Attacks on GGH13-based obfuscation

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ENS de Lyon

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Indistinguishability Obfuscation (iO)

$$C \equiv C'$$

$C(x) = C'(x)$ for all x

$$\Downarrow$$

$$\mathcal{O}(C) \approx_c \mathcal{O}(C')$$

3 categories

- Branching program obfuscation
- Circuit obfuscation
- Obfuscation from functional encryption

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Overview of the talk

1. Simple obfuscator
2. Quantum attack
3. State-of-the-art

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1. Simple obfuscator
2. Quantum attack
3. State-of-the-art

Branching programs

A branching program represents a function (cf Turing machine, or circuit).

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A Branching Program (BP) is a collection of

- 2ℓ matrices $M_{i,b}$ (for $i \in \{1, \ldots, \ell\}$ and $b \in \{0, 1\}$),
- two vectors M_0 and $M_{\ell+1}$,
- a vector inp $\in \{1, \ldots, r\}^\ell$ (where r is the size of the input).

BP

	x_1	x_1	x_2	x_1	x_3	x_2	
M_0	$M_{1,1}$ $M_{1,0}$	$M_{2,1}$ $M_{2,0}$	$M_{3,1}$ $M_{3,0}$	$M_{4,1}$ $M_{4,0}$	$M_{5,1}$ $M_{5,0}$	$M_{6,1}$ $M_{6,0}$	M_7

Evaluation on $x = 0 \quad 1 \quad 1$

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Cryptographic multilinear maps (asymmetric setting)

Different levels of encodings, parametrized by sets $S \subseteq \{1, \cdots, \kappa\}$.

Definition: asymmetric multilinear map

$\mathsf{Enc}(a, S)$: encoding of a at level S.
$S^* = \{1, \cdots, \kappa\}$, maximum level.

Addition: $\mathsf{Add}(\mathsf{Enc}(a_1, S), \mathsf{Enc}(a_2, S)) = \mathsf{Enc}(a_1 + a_2, S)$.

Multiplication: if $S_1 \cap S_2 = \emptyset$,

$$\mathsf{Mult}(\mathsf{Enc}(a_1, S_1), \mathsf{Enc}(a_2, S_2)) = \mathsf{Enc}(a_1 \cdot a_2, S_1 \cup S_2).$$

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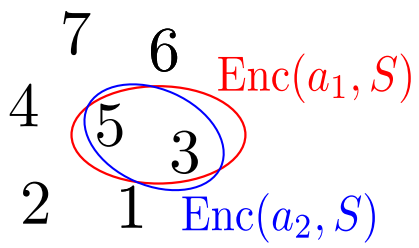

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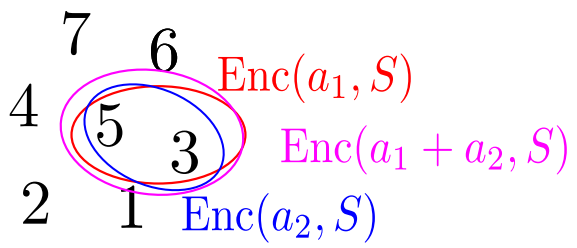

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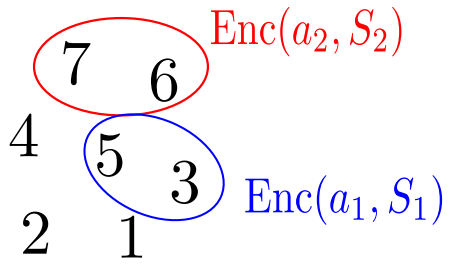

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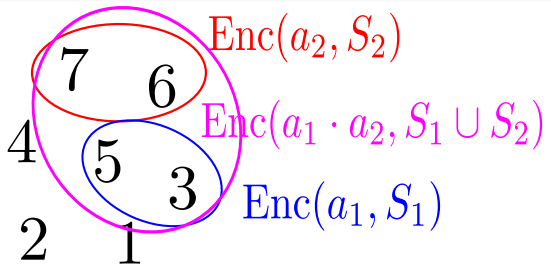

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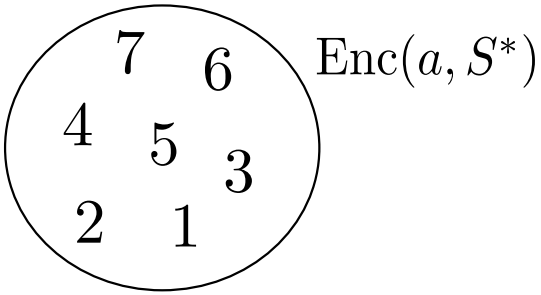
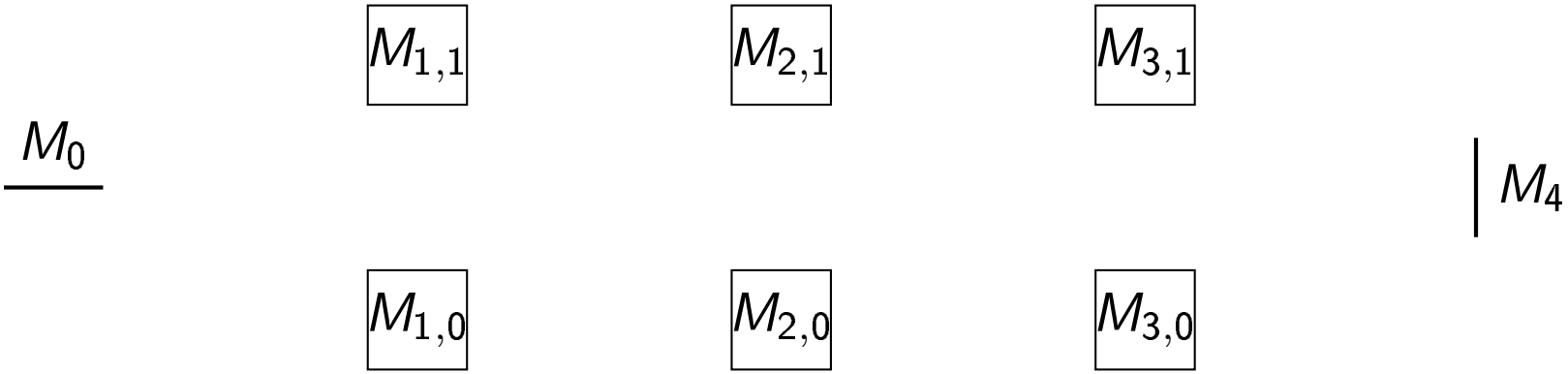


Simple obfuscator

[GGH+13b, BR14, BGK+14, PST14, AGIS14, MSW14, GMM+16]

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using mmap
- **Output:** The encoded matrices and vectors



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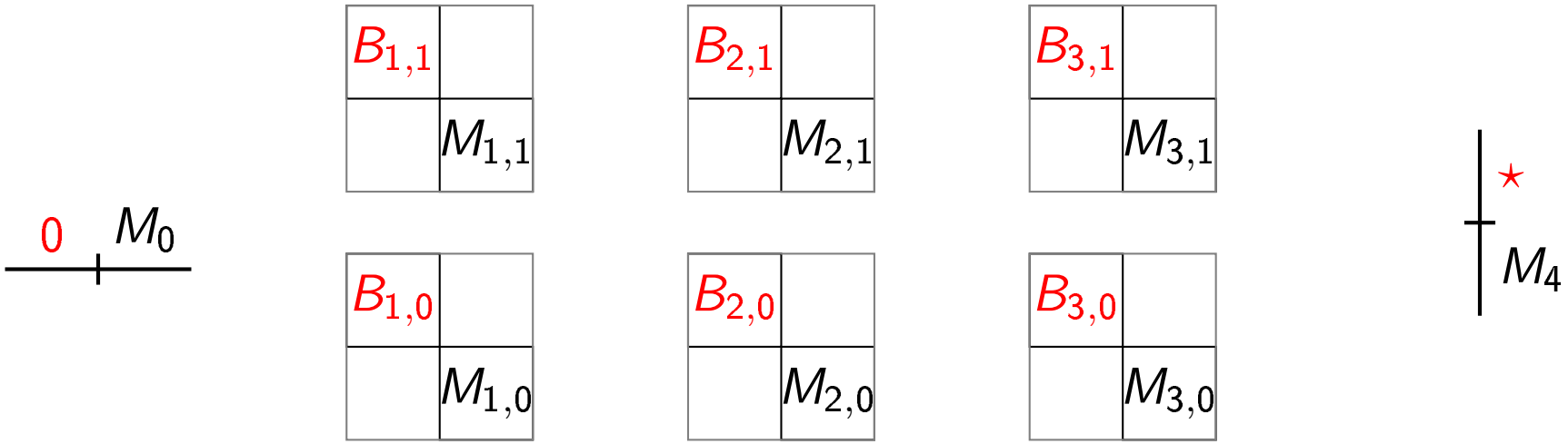

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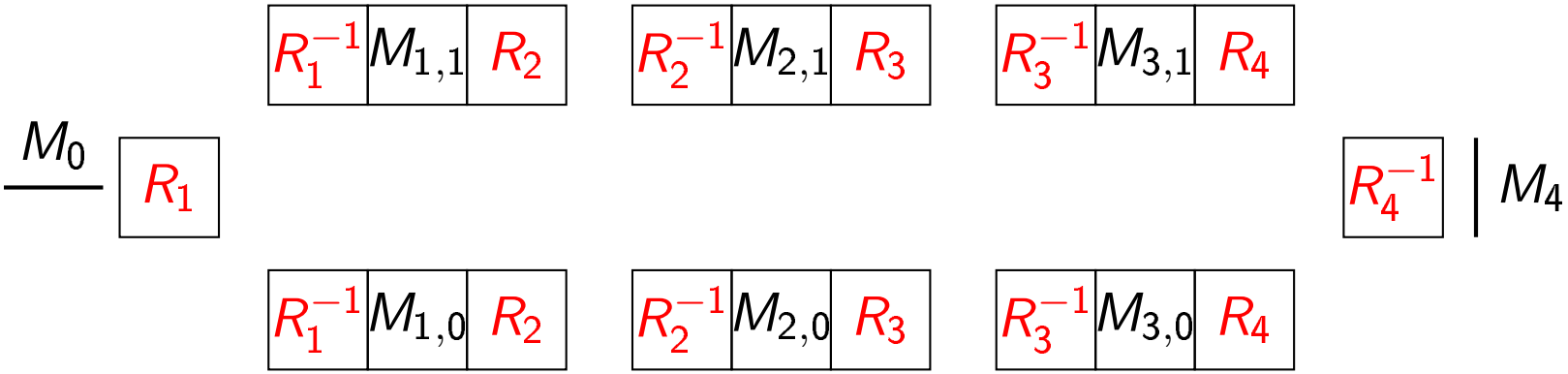

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$$\underline{M_0} \qquad \begin{matrix} \alpha_{1,1} \times \boxed{M_{1,1}} & \alpha_{2,1} \times \boxed{M_{2,1}} & \alpha_{3,1} \times \boxed{M_{3,1}} \\ \alpha_{1,0} \times \boxed{M_{1,0}} & \alpha_{2,0} \times \boxed{M_{2,0}} & \alpha_{3,0} \times \boxed{M_{3,0}} \end{matrix} \qquad \Big| \, M_4$$

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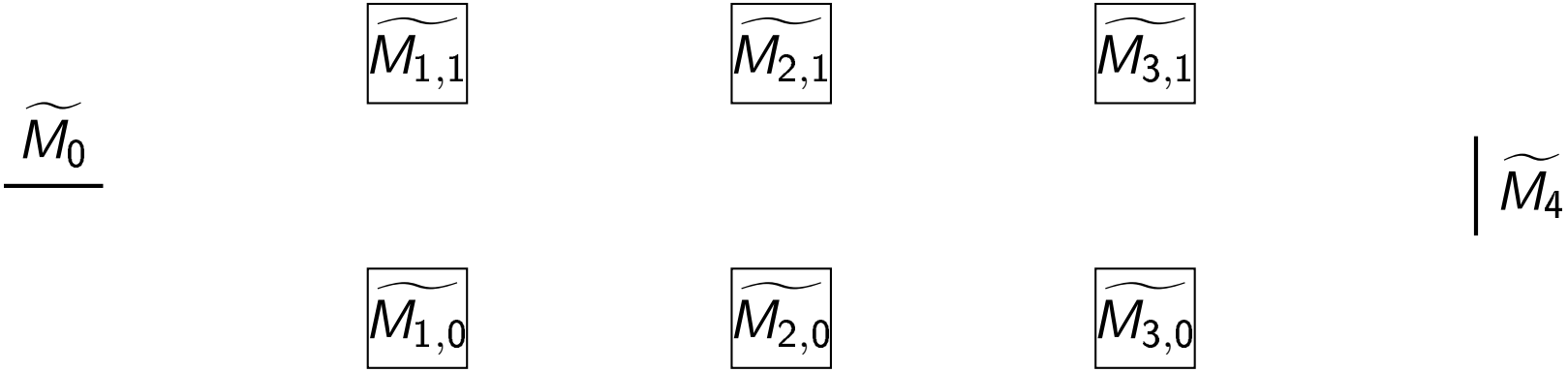

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$S^* = \{1, 2, 3, 4, 5\}$

$\mathrm{Enc}(\boxed{\widetilde{M_{1,1}}}, \{2\})$ $\mathrm{Enc}(\boxed{\widetilde{M_{2,1}}}, \{3\})$ $\mathrm{Enc}(\boxed{\widetilde{M_{3,1}}}, \{4\})$

$\mathrm{Enc}(\underline{\widetilde{M_0}}, \{1\})$

$\mathrm{Enc}(\widetilde{M_4}, \{5\})$

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Outline of the talk

GGH13 in a quantum world

Reminder: asymmetric multilinear map

$\mathrm{Enc}(a, S)$ = encoding of a at level S.
$S^* = \{1, \cdots, \kappa\}$ maximum level.

Addition: $\mathrm{Add}(\mathrm{Enc}(a_1, S), \mathrm{Enc}(a_2, S)) = \mathrm{Enc}(a_1 + a_2, S)$.

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e.g. $\{1, 3, 4\} \uplus \{2, 3\} = \{1, 2, 3, 3, 4\}$

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$2S^* = S^* \uplus S^* = \{1, 1, 2, 2, \cdots, \kappa, \kappa\}$

With a quantum computer

$$\text{Double-zero-test}(\mathrm{Enc}(a, 2S^*)) = \text{True iff } a = 0 \bmod p^2$$

Mixed-input attack

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- $M_{i,b}$ input branching program
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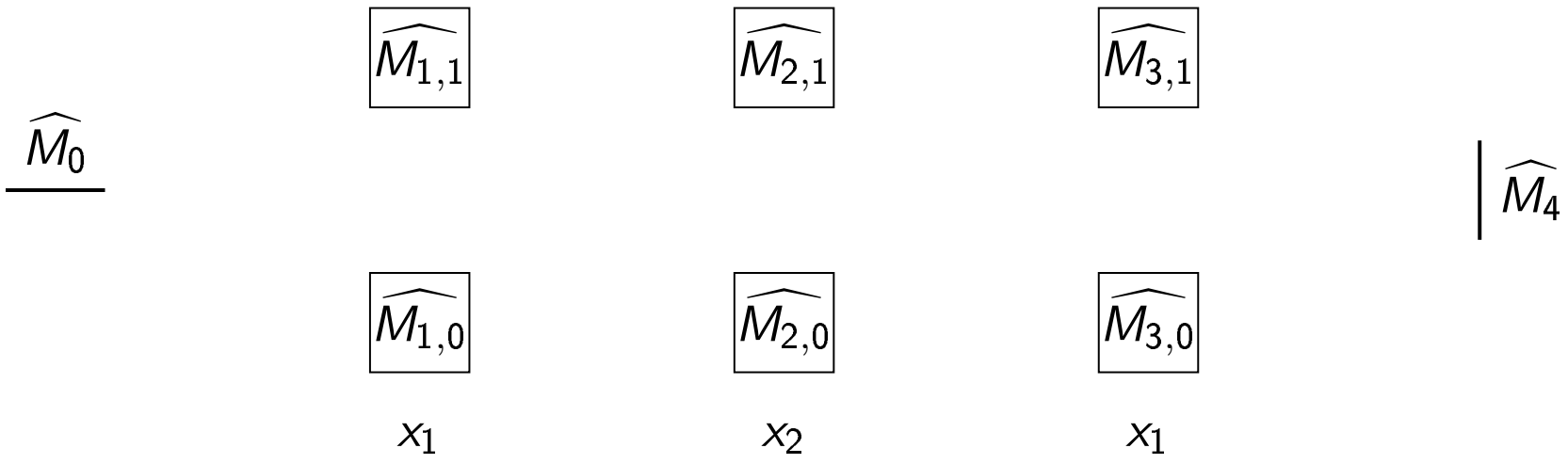

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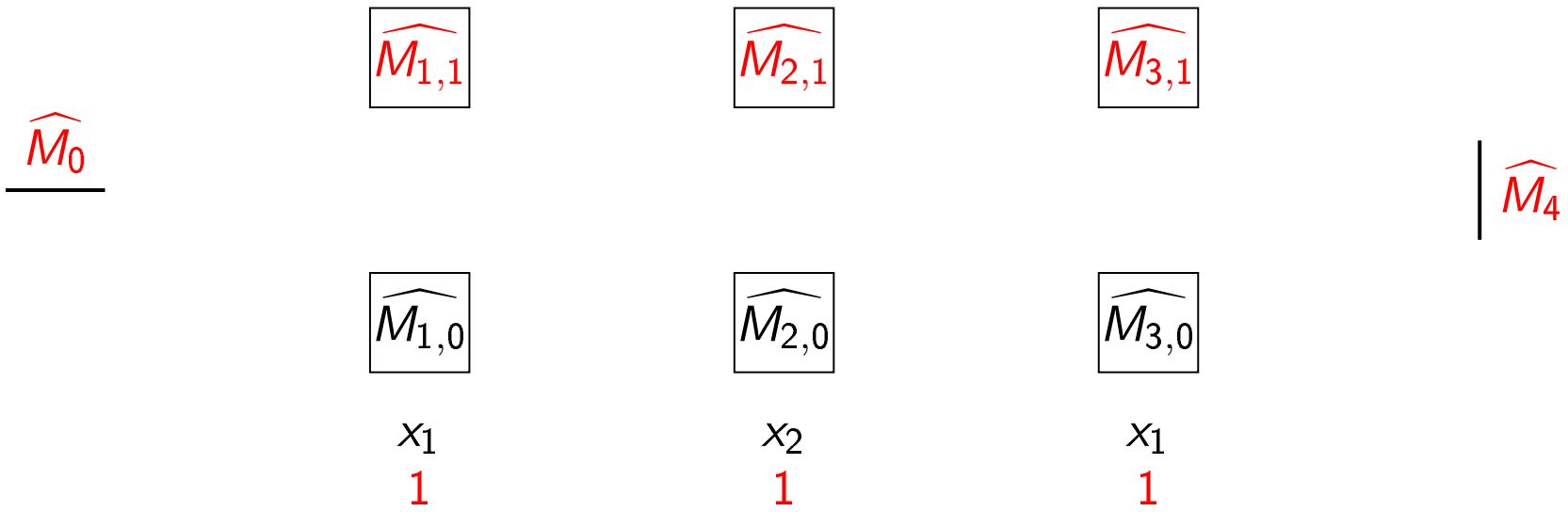

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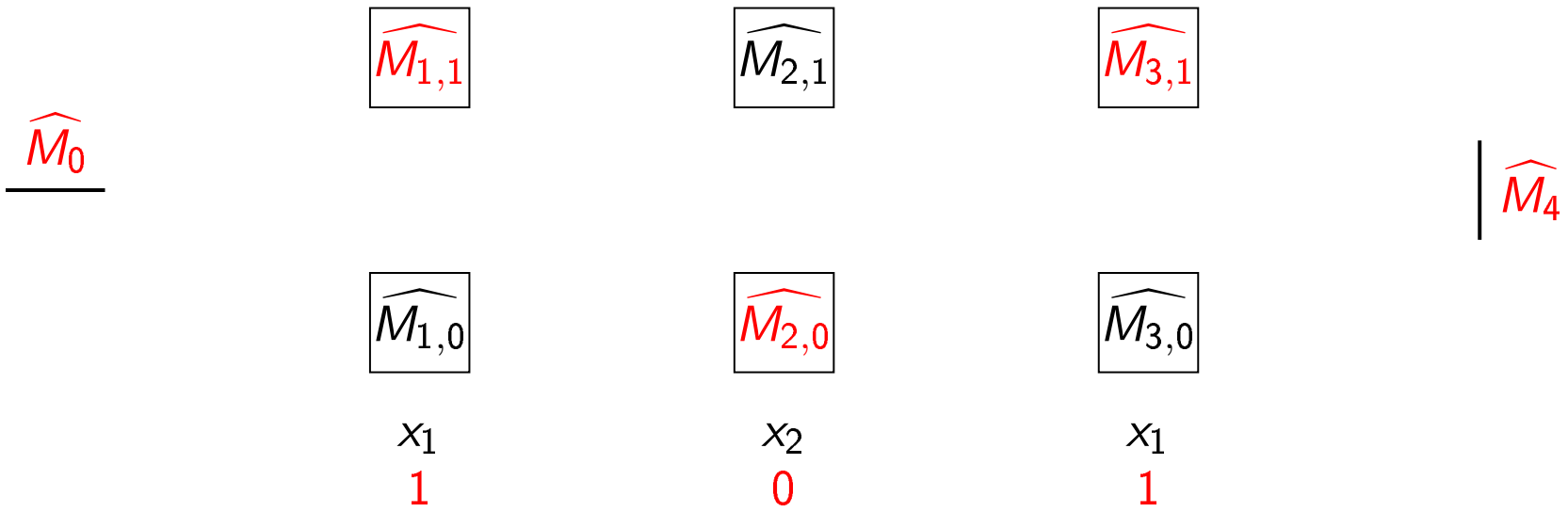

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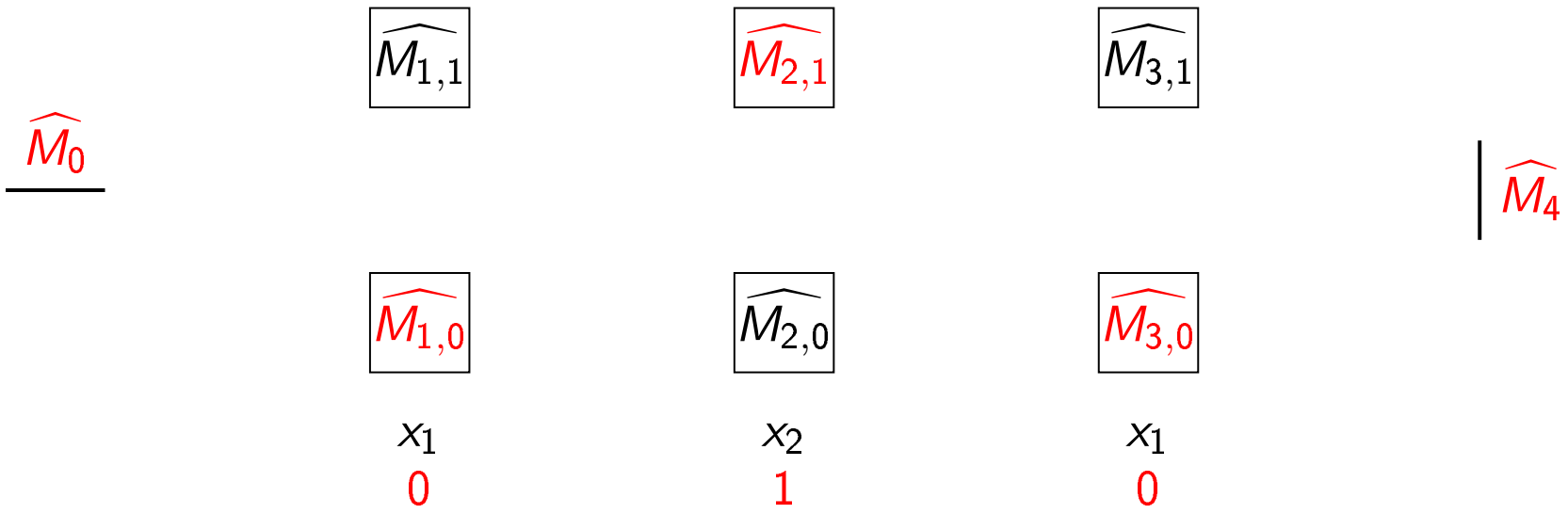

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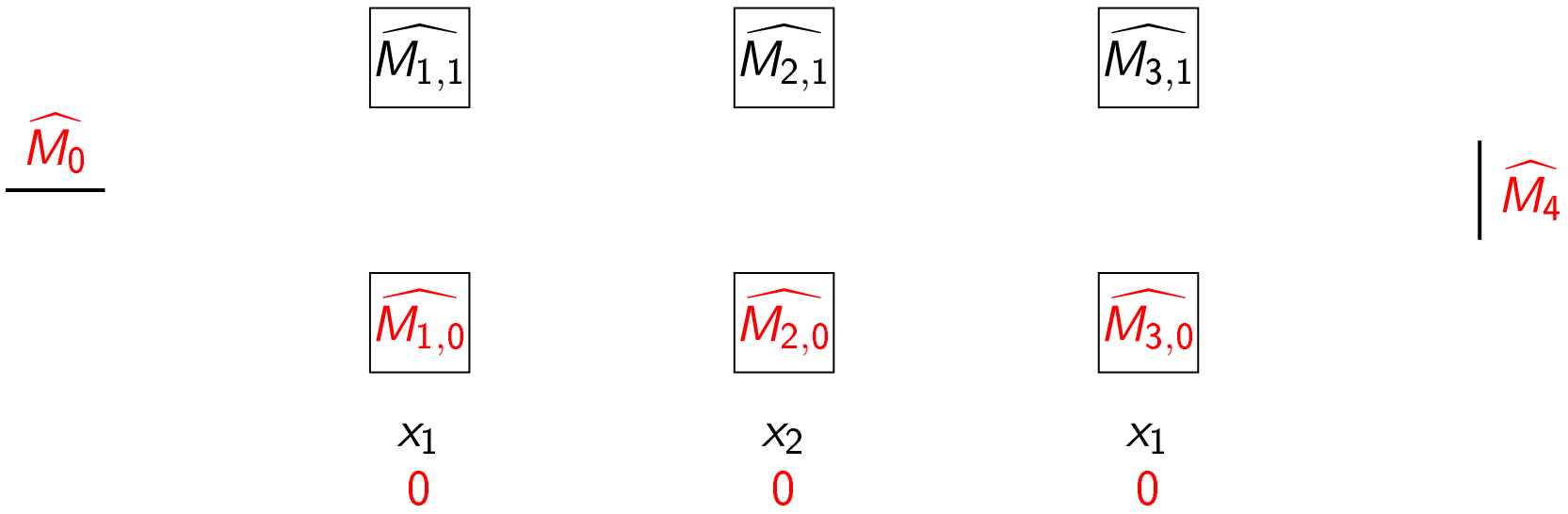

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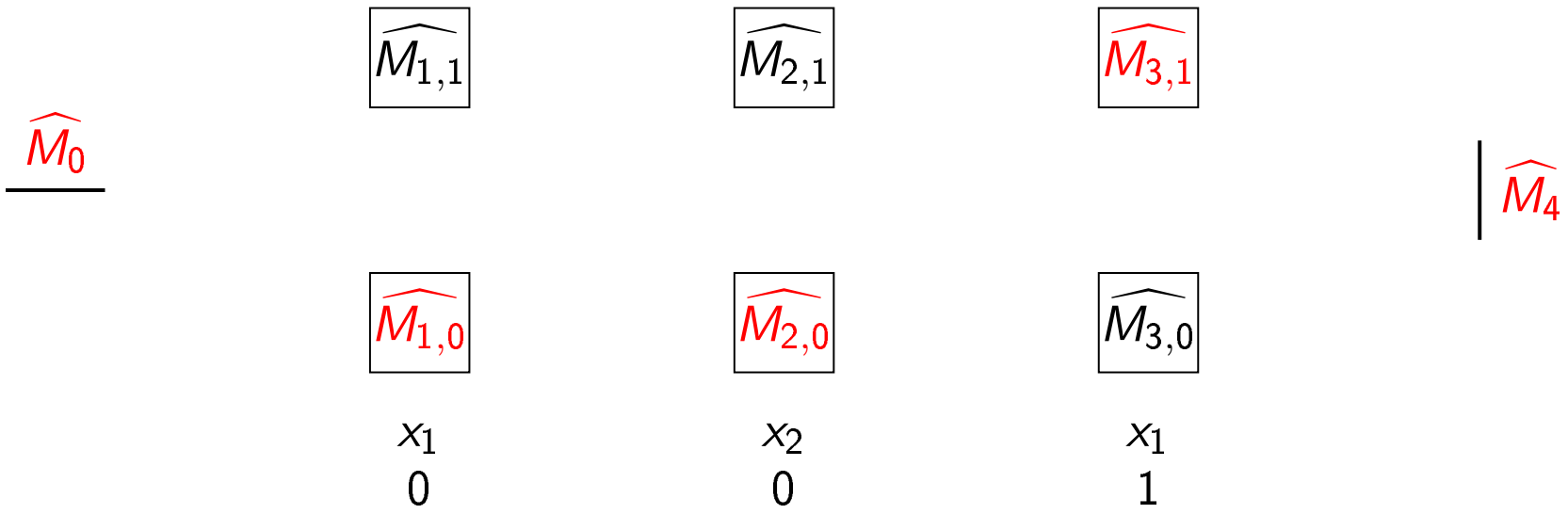

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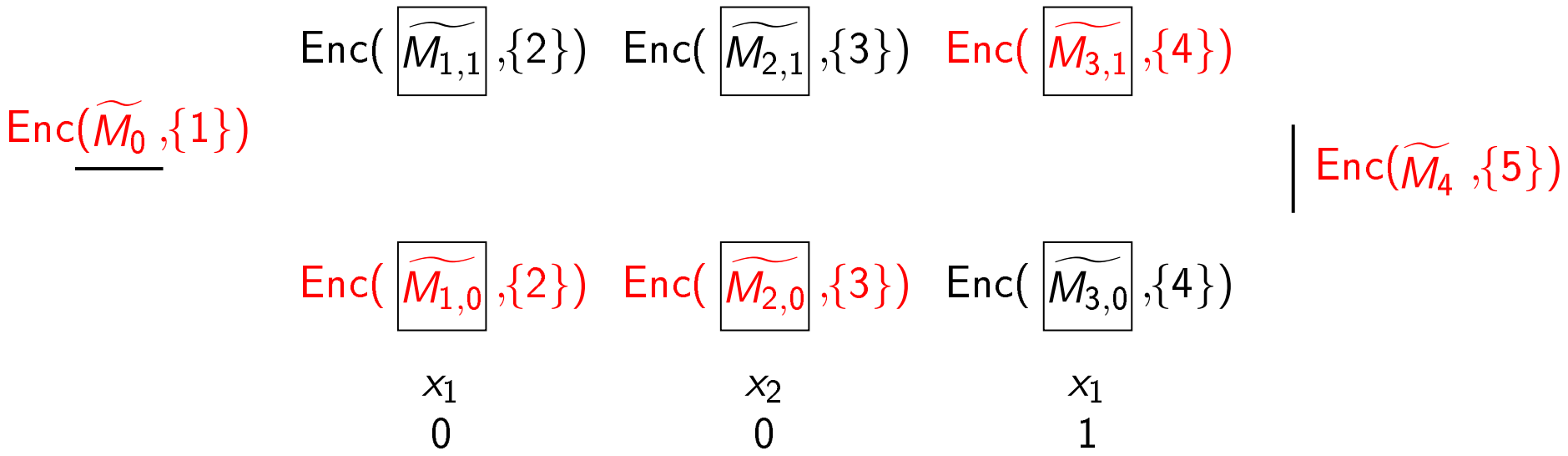

Preventing mixed-input attacks

- In the randomization phase $\Rightarrow$ not in this talk [GGH+13b, BR14]

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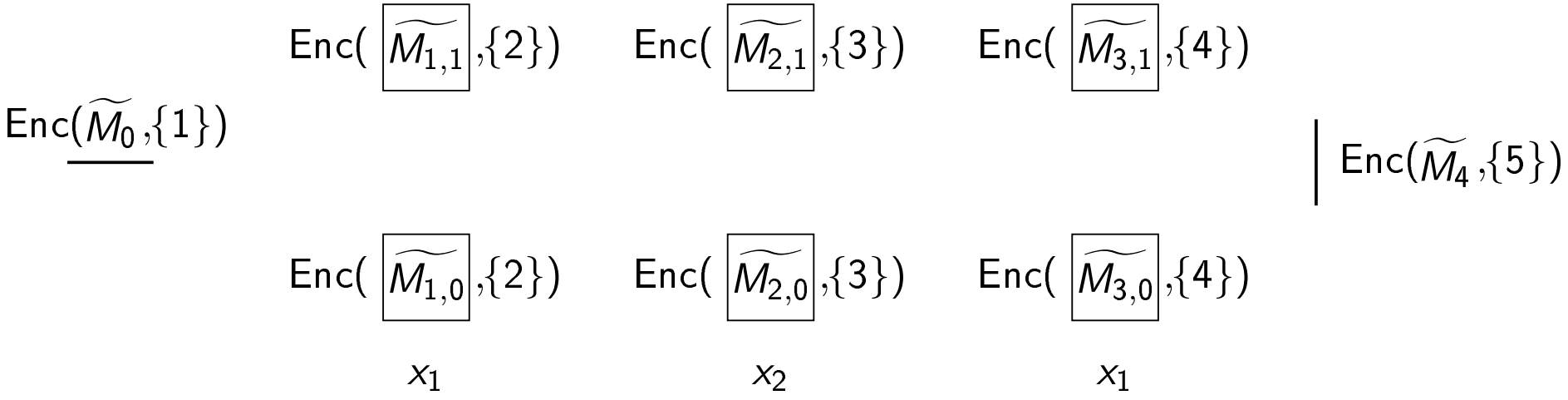

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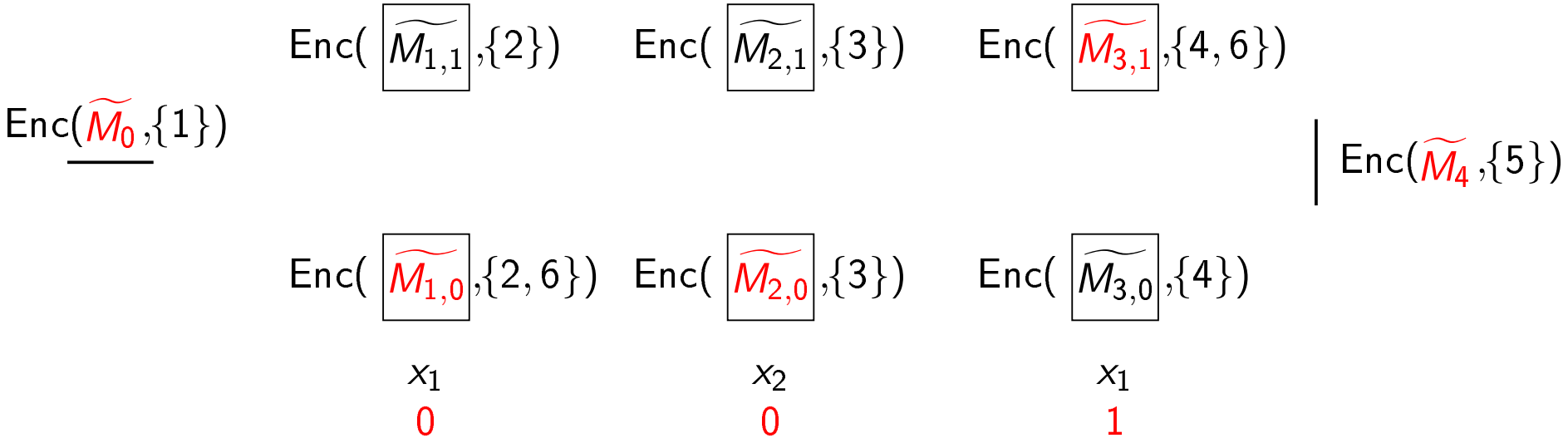

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0	0	1

Total level: $\{1, 2, 3, 4, 5, 6, 6\}$ ⇒ cannot zero-test

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Generalisation: $\{1\}, \{2, 3\}, \{4, 5\}, \{6, 7\}$
$\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7\}$

Attack idea: double mixed input

Reminder

In quantum world, we have

$$\text{Double-zero-test}(\text{Enc}(a, 2S^*)) = \text{True iff } a = 0 \bmod p^2$$

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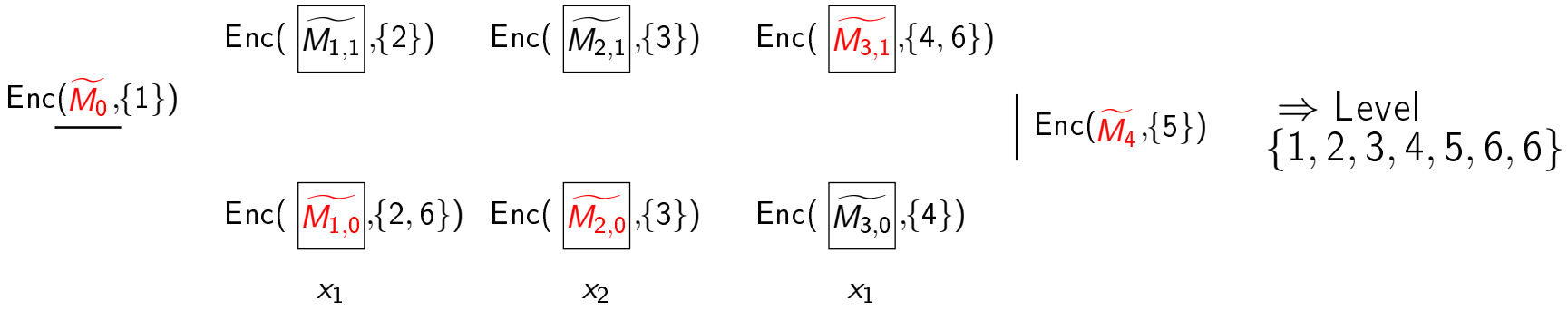

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Product level: $S^* \uplus S^* = 2S^*$

iO distinguishing attack

Reminder: iO

$$\forall C_1 \equiv C_2,\ \mathcal{O}(C_1) \simeq_c \mathcal{O}(C_2)$$

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Objective: Find $C_1 \equiv C_2$ s.t. double mixed input product is 0 on C_1 and $\neq 0$ on C_2, e.g.

- the two mixed-input are 0 mod p for C_1
  $\Rightarrow$ product is 0 mod p^2
- the two mixed-input are $\neq 0$ mod p for C_2
  $\Rightarrow$ product is $\neq 0$ mod p^2

One example of C_1 and C_2

$$\boldsymbol{C_1}: \quad \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{array}{ccc} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ x_1 & x_2 & x_1 \end{array} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \Rightarrow \forall x,\ C_1(x) = 0$$

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$$\boxed{\text{We can distinguish } \mathcal{O}(C_1) \text{ from } \mathcal{O}(C_2)}$$

Outline of the talk

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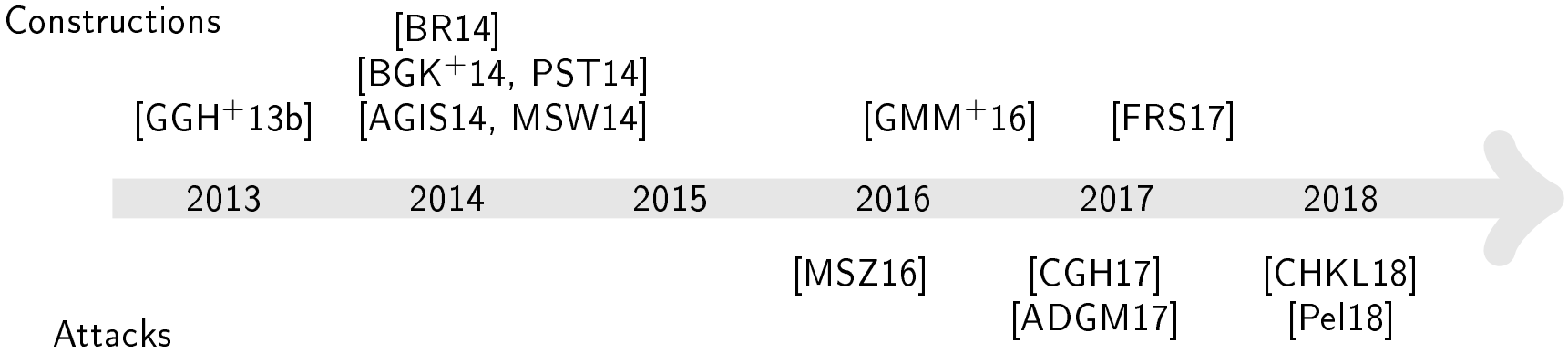

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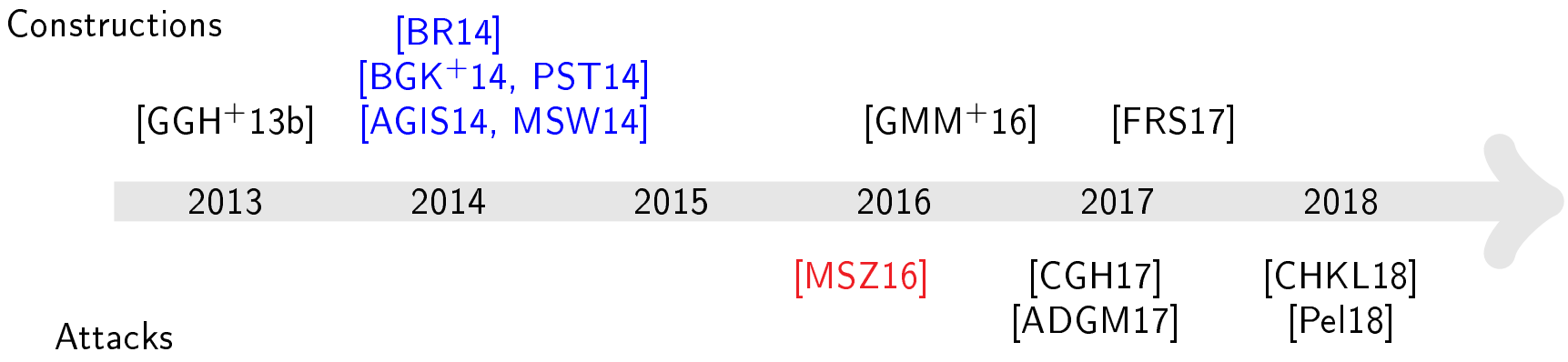

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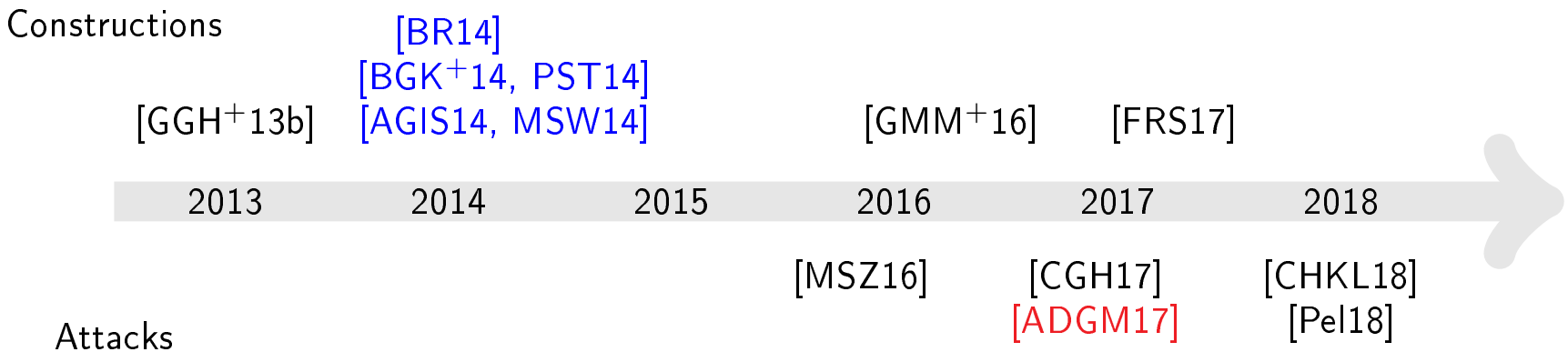

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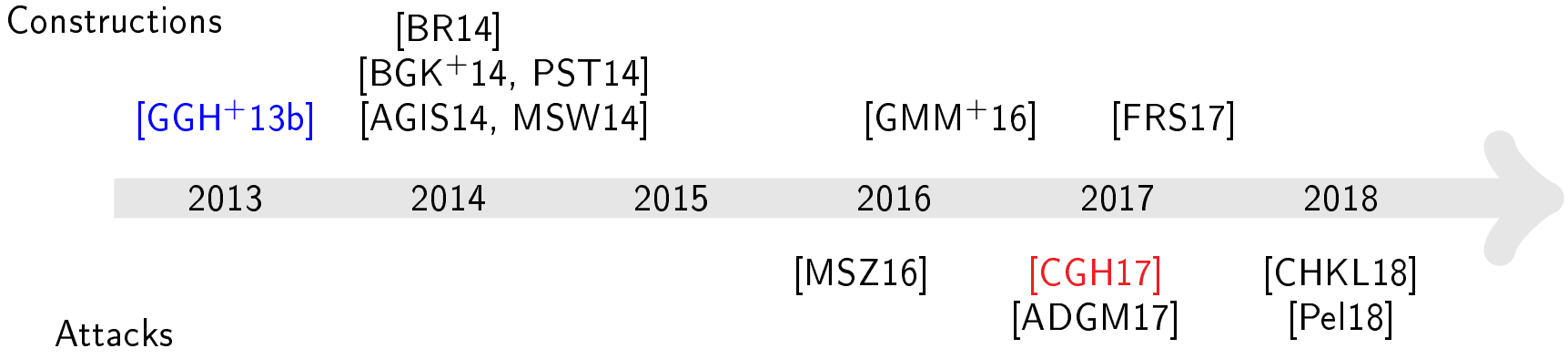

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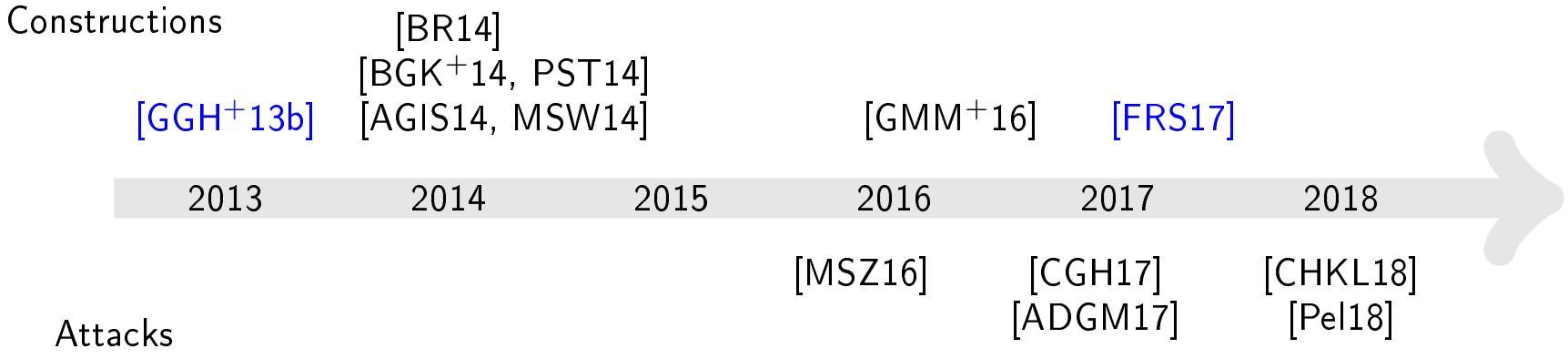

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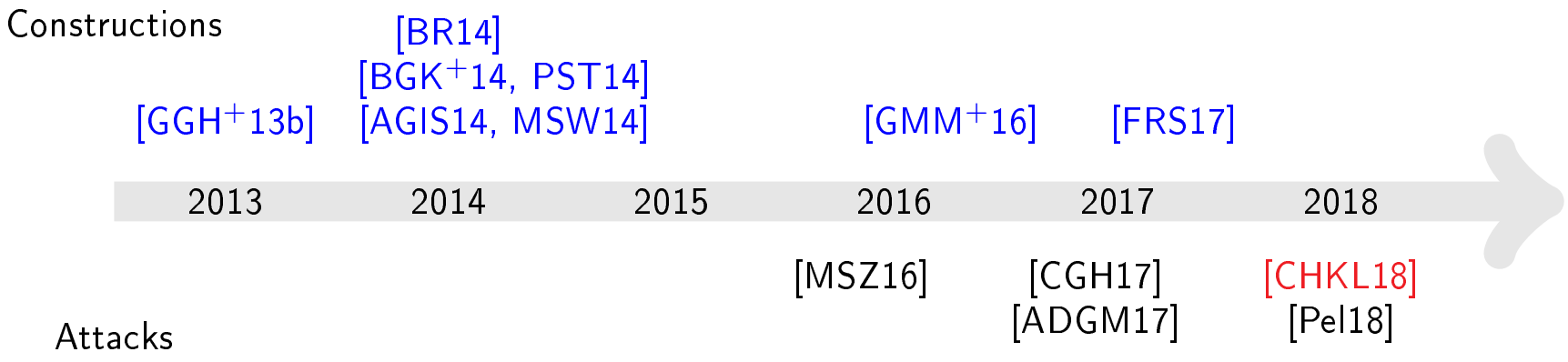

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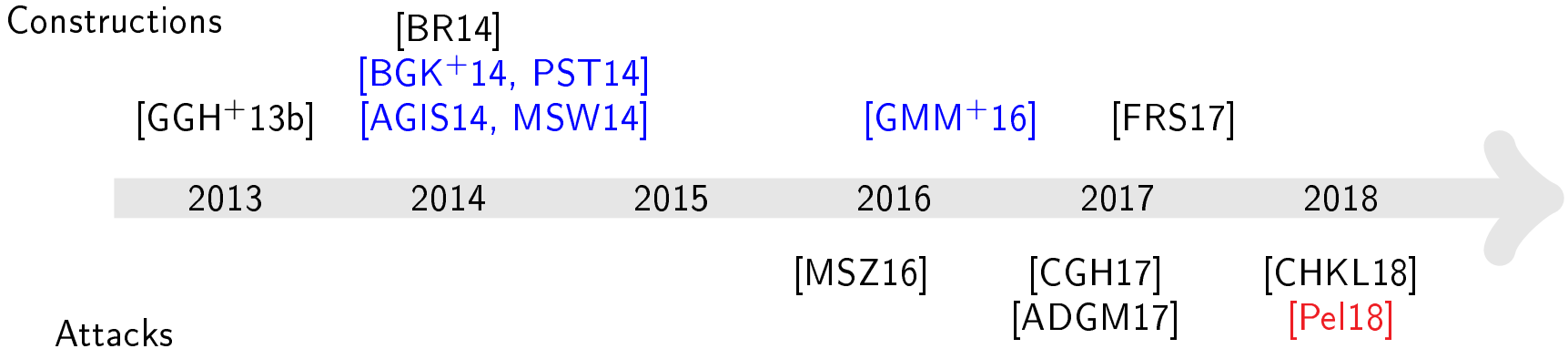
[CHKL18]: NTRU attack for specific choices of parameters

History (GGH13-based branching program obfuscation)



[MSZ16]: all constructions without diagonal blocks

[ADGM17]: idem MSZ but from circuits

[CGH17]: use input-partitionability (cf CLT13) $\Rightarrow$ prevented by [FRS17]

[CHKL18]: NTRU attack for specific choices of parameters

[Pel18]: quantum attack

Current status

Attacks \ iOs	[GGH+13b]	[BR14, BGK+14, PST14, AGIS14, MSW14]	[GMM+16]	circuit obfuscators [Zim15, AB15, DGG+18]
[MSZ16]		fully broken		
[CGH17]	input-partitionable			
[CHKL18]	some parameters		some parameters	
[Pel18]			quantum	quantum

Still standing classically:

- [GGH+13b]+[FRS17]
- [GMM+16]
- all circuit obfuscators

Still standing quantumly:

- [GGH+13b]+[FRS17]

Current status

Attacks \ iOs	[GGH+13b]	[BR14, BGK+14, PST14, AGIS14, MSW14]	[GMM+16]	circuit obfuscators [Zim15, AB15, DGG+18]
[MSZ16]		fully broken		
[CGH17]	input-partitionable			
[CHKL18]	some parameters		some parameters	
[Pel18]			quantum	quantum

Still standing classically:

- [GGH+13b]+[FRS17]
- [GMM+16]
- all circuit obfuscators

Still standing quantumly:

- [GGH+13b]+[FRS17]

Questions?

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