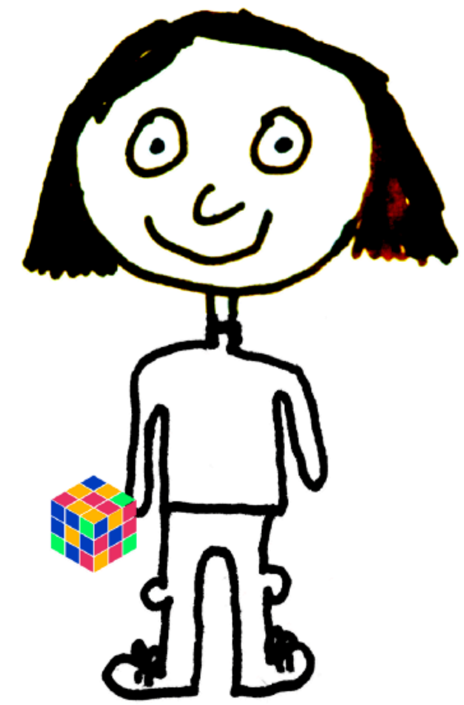
Multi-Theorem Preprocessing NIZKs from Lattices

Sam Kim and David J. Wu
Stanford University

Zero-Knowledge Proofs [GMR85]

NP Language $\mathcal{L}$

Prover(x, w)



Verifier(x)

Zero-Knowledge Proofs [GMR85]

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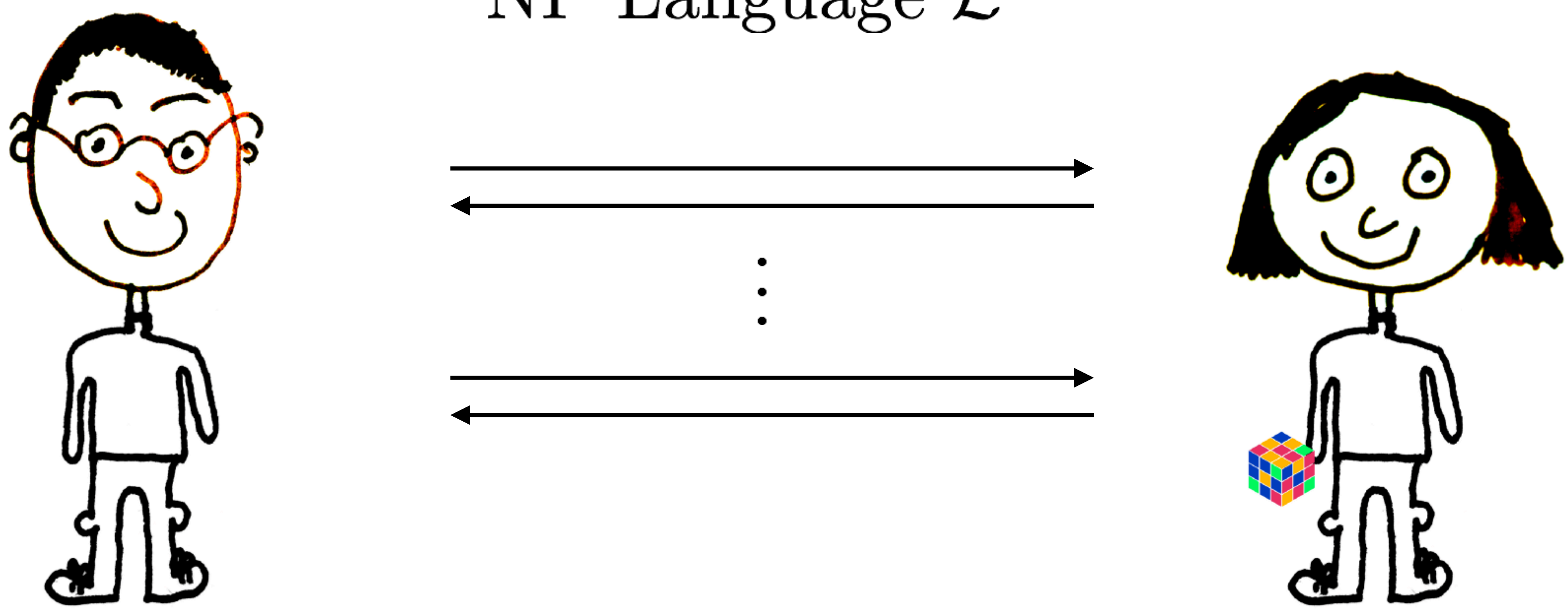

Prover(x, w) Verifier(x)

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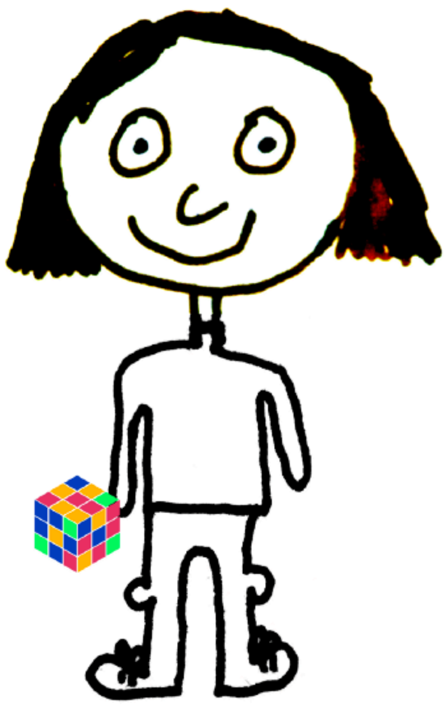

Verifier(x)

Accept / Reject

Zero-Knowledge Proofs [GMR85]

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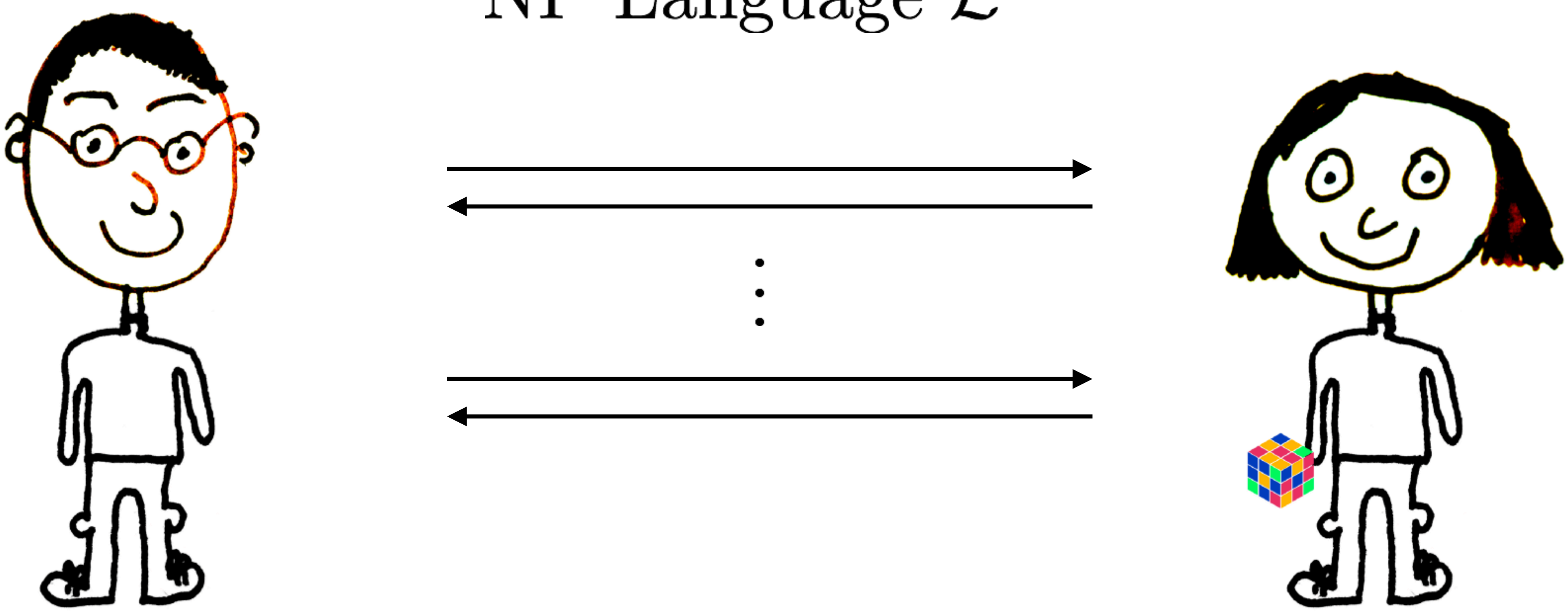

Prover(x, w)

Verifier(x)

$\downarrow$

Accept

Requirements:

1. Completeness

Zero-Knowledge Proofs [GMR85]

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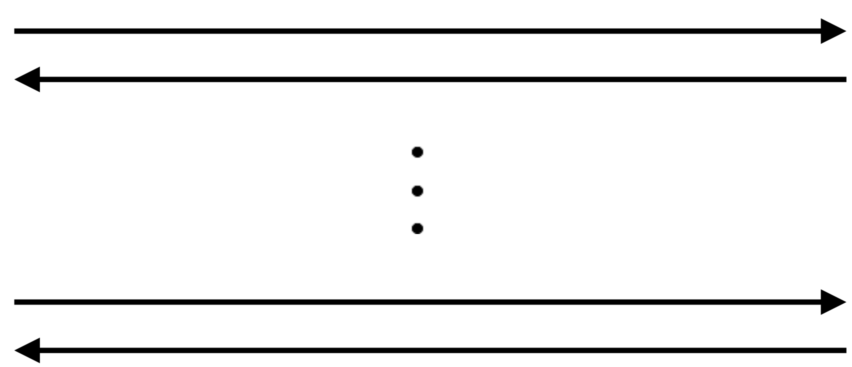
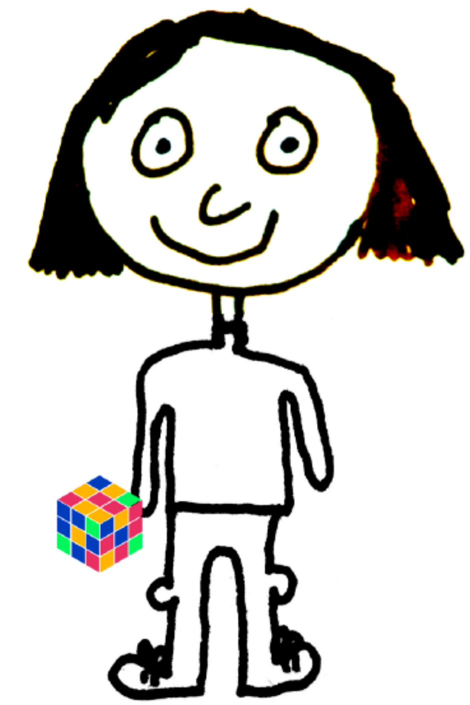




Prover$(x \notin \mathcal{L})$

Verifier(x)

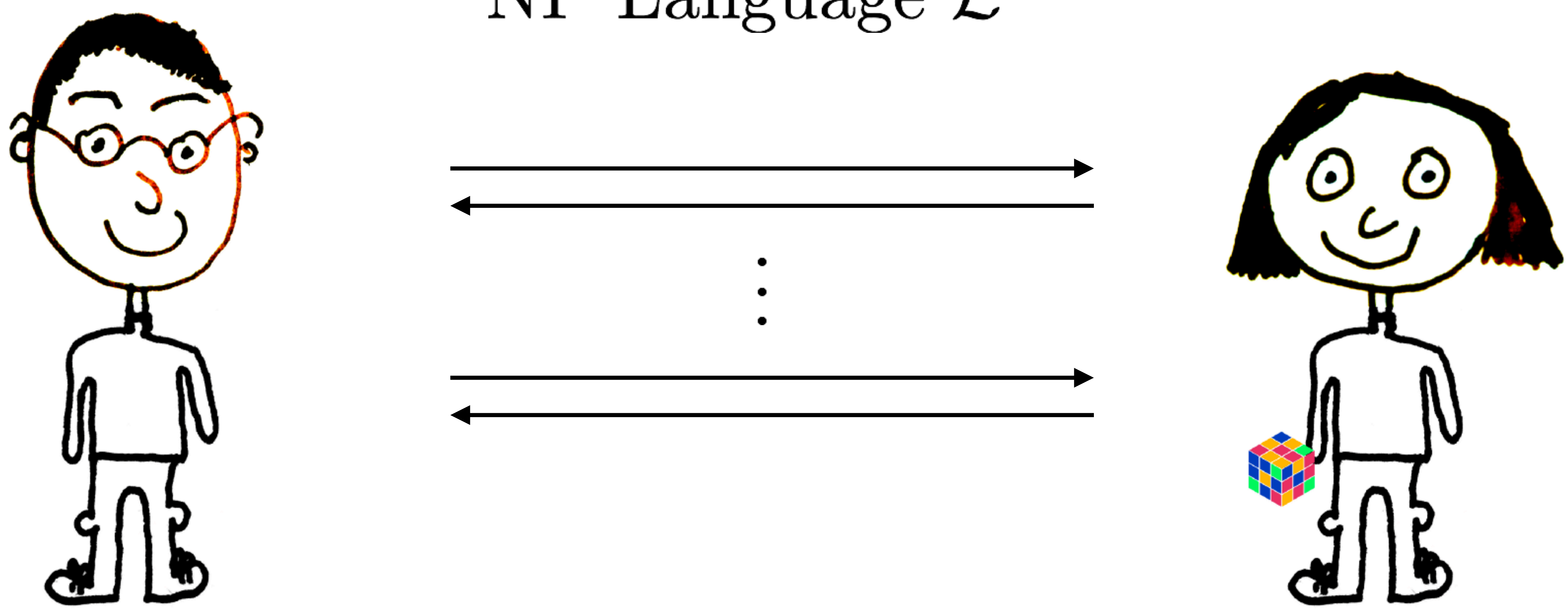
$\downarrow$

Reject

Requirements:
1. Completeness
2. Soundness

Zero-Knowledge Proofs [GMR85]

NP Language $\mathcal{L}$



Prover(x, w) Verifier(x)

Requirements:

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2. Soundness
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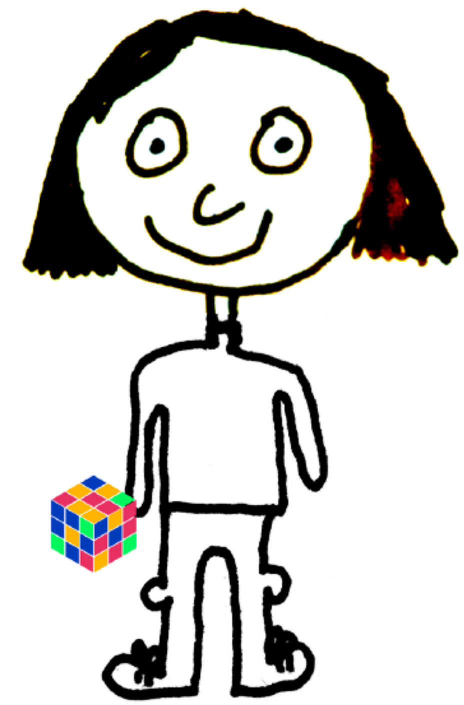

$\mathsf{Sim}(x \in \mathcal{L})$

$\mathsf{Verifier}(x)$

$\downarrow$

???

Requirements:

1. Completeness
2. Soundness
3. Zero-Knowledge

Non-Interactive Zero-Knowledge (NIZK)

Natural to ask: Can we have “one-shot” ZK proofs?

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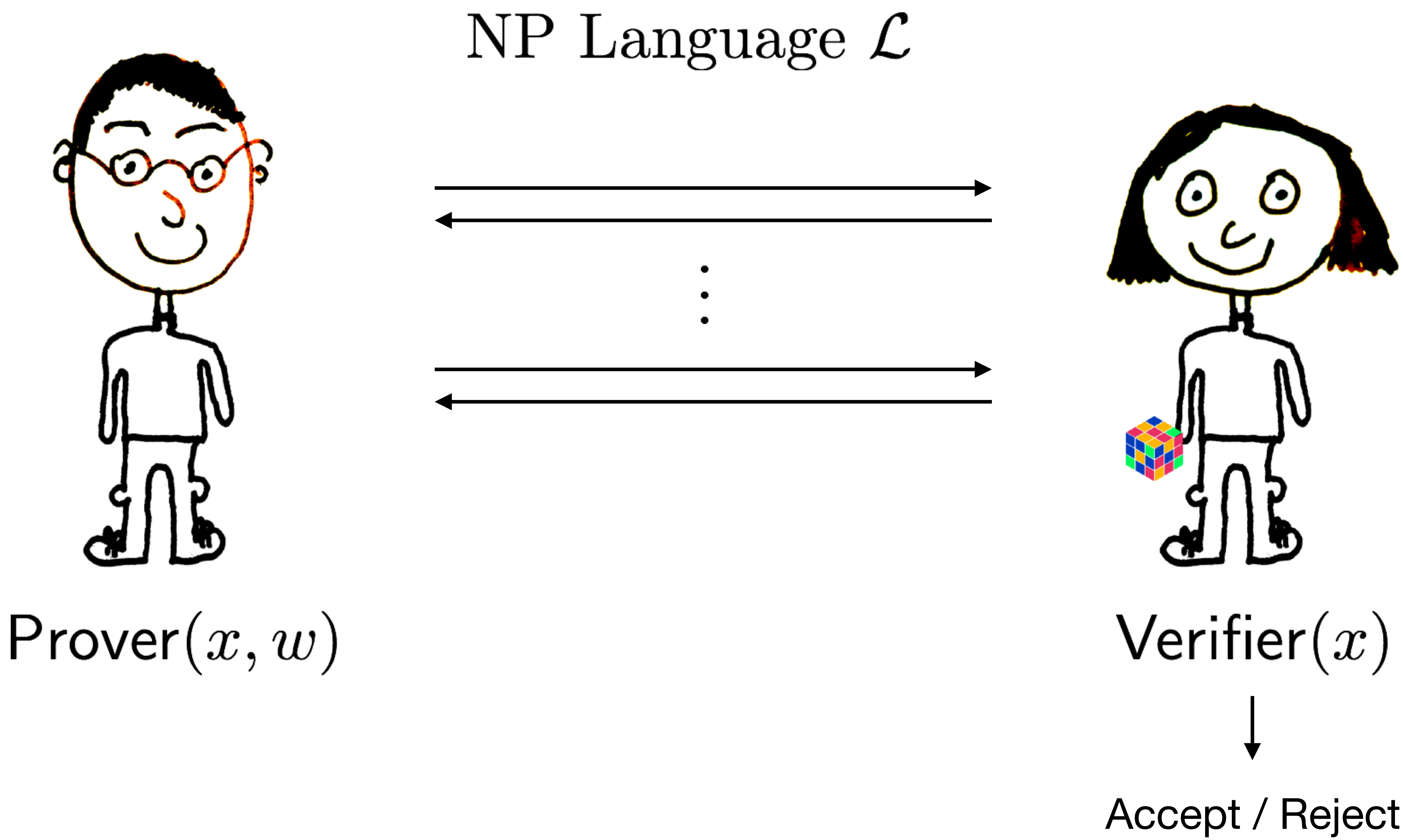
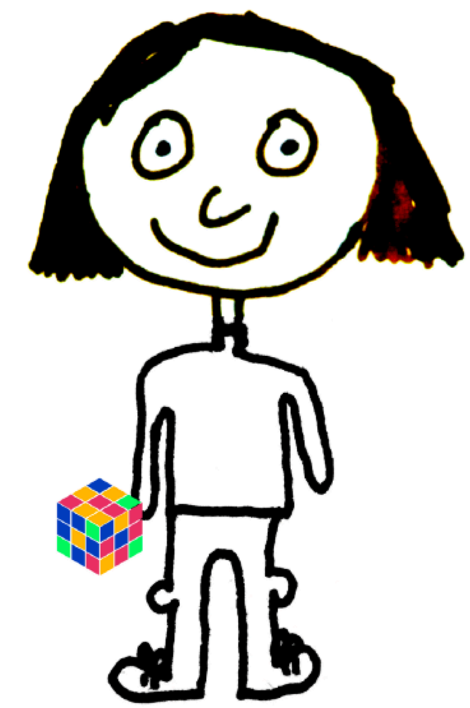


Non-Interactive Zero-Knowledge (NIZK)

Natural to ask: Can we have "one-shot" ZK proofs?

$\text{NP Language } \mathcal{L}$

$$\xrightarrow{\quad \pi \quad}$$



Prover(x, w)

Verifier(x)

$\downarrow$

Accept / Reject

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Can only achieve for “easy” languages in standard model [GO94]

Soundness + **Zero-Knowldge** implies Efficient Decision Algorithm

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Work with **weaker models**:

- Random Oracle Model
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This Work: We focus on the **CRS Model** (or **preprocessing model**)

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Constructions for all of NP ?

(w/ efficient provers, reusable CRS, publicly verifiable, …)

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Still no construction from LWE.

- NIZK for specific languages [PV08, APS18, RSS18, …]

Our Results

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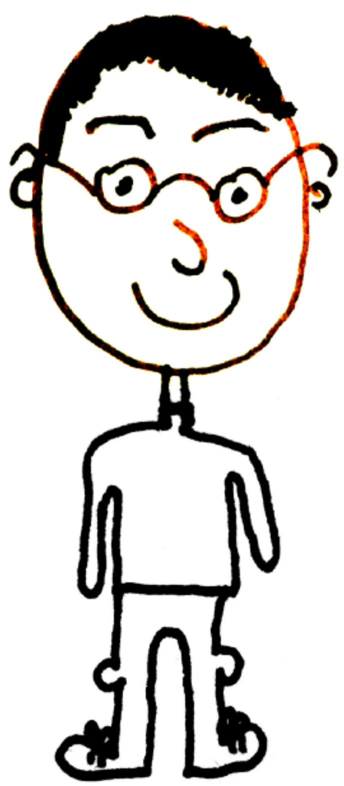
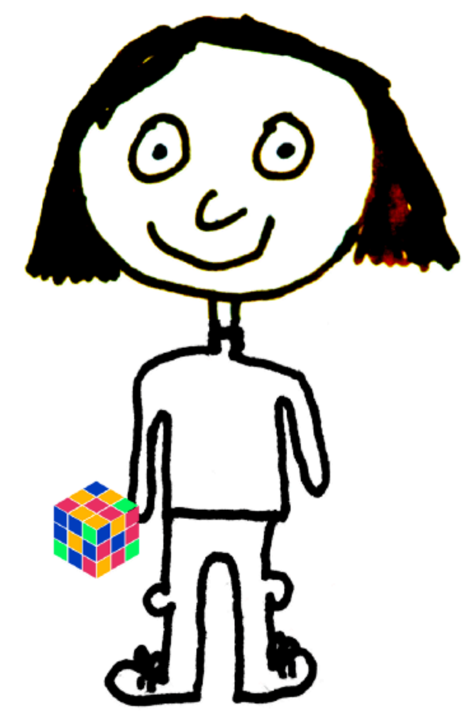
3. Applications to MPC

NIZK with Preprocessing [DMP88]

NP Language $\mathcal{L}$

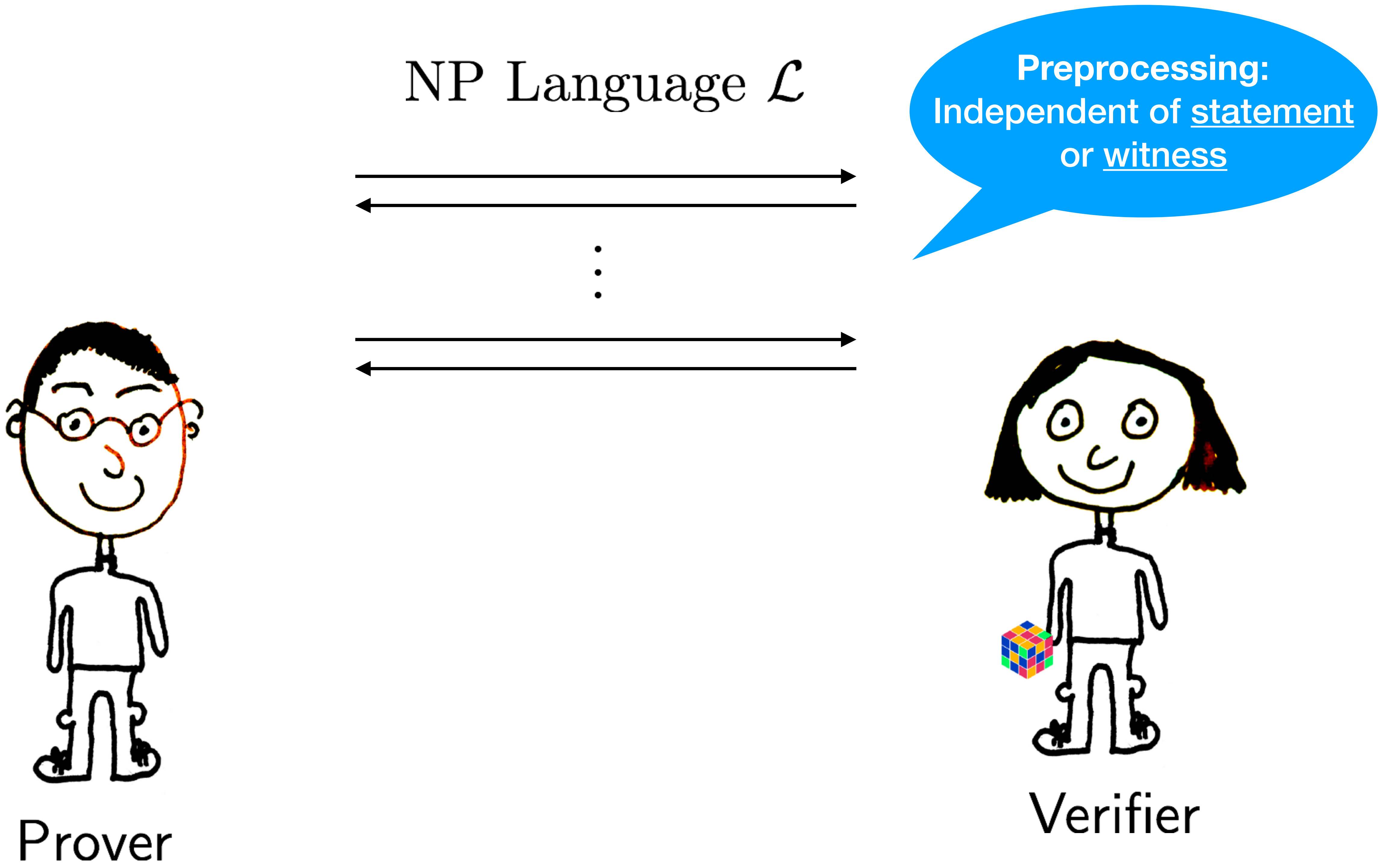


Prover



Verifier

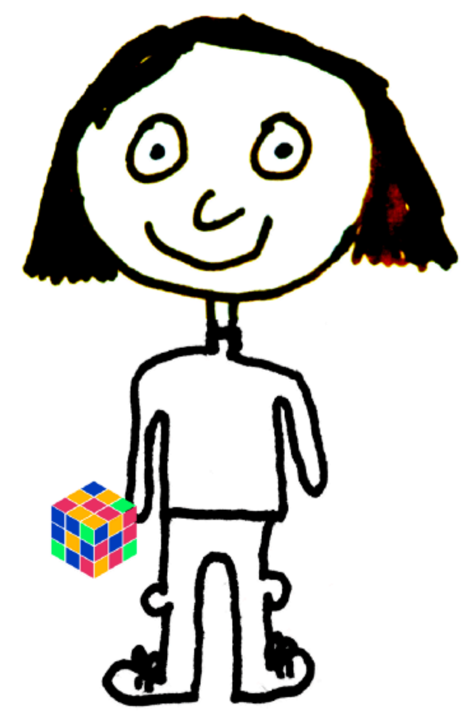
NIZK with Preprocessing [DMP88]



NIZK with Preprocessing [DMP88]

NP Language $\mathcal{L}$

Prover
k_P



Verifier
k_V

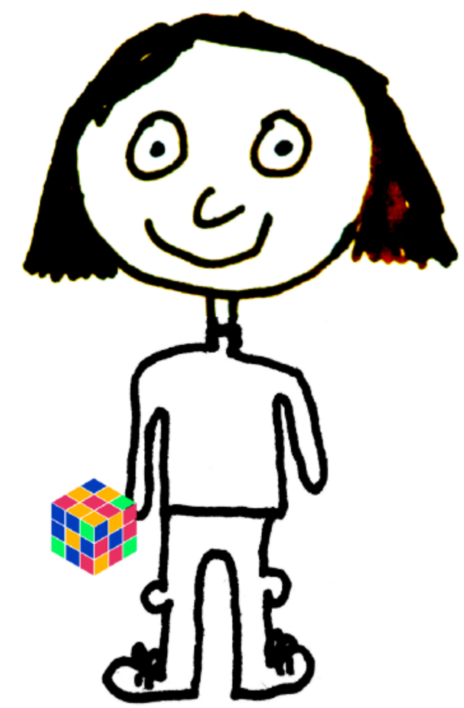
NIZK with Preprocessing [DMP88]

NP Language $\mathcal{L}$

Prover

k_P

$$\xrightarrow{\pi_x}$$

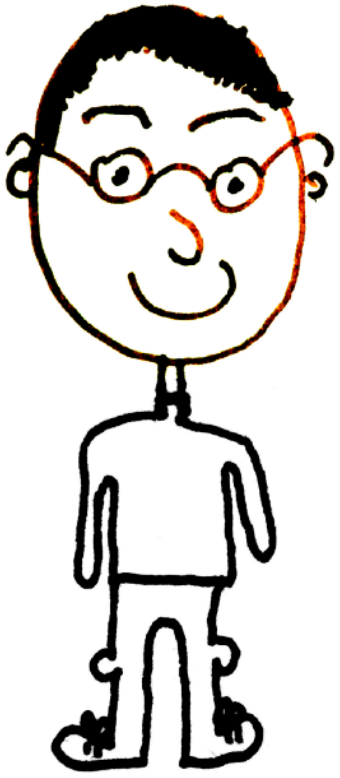


Verifier

k_V

NIZK with Preprocessing [DMP88]
(Single-Theorem)

NP Language $\mathcal{L}$



Prover
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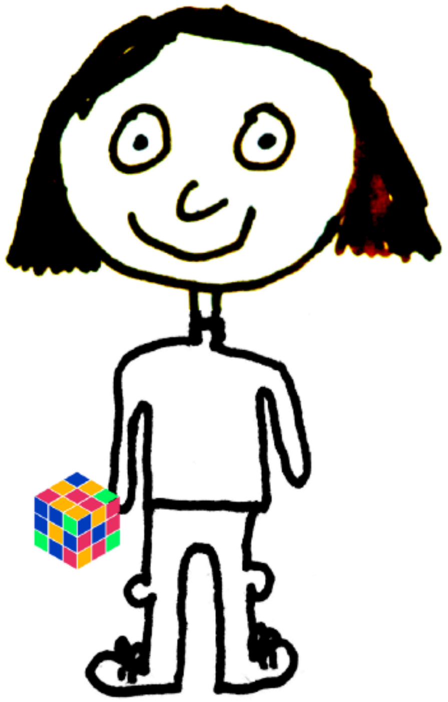

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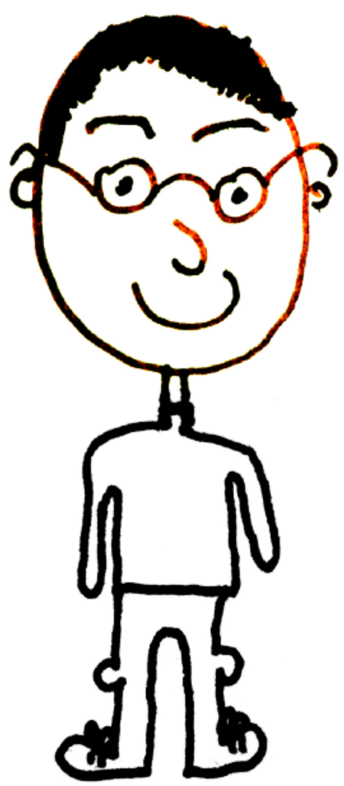

Prover
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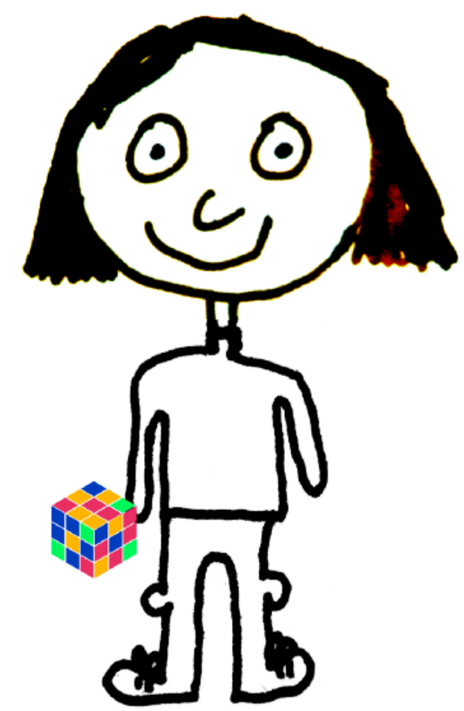

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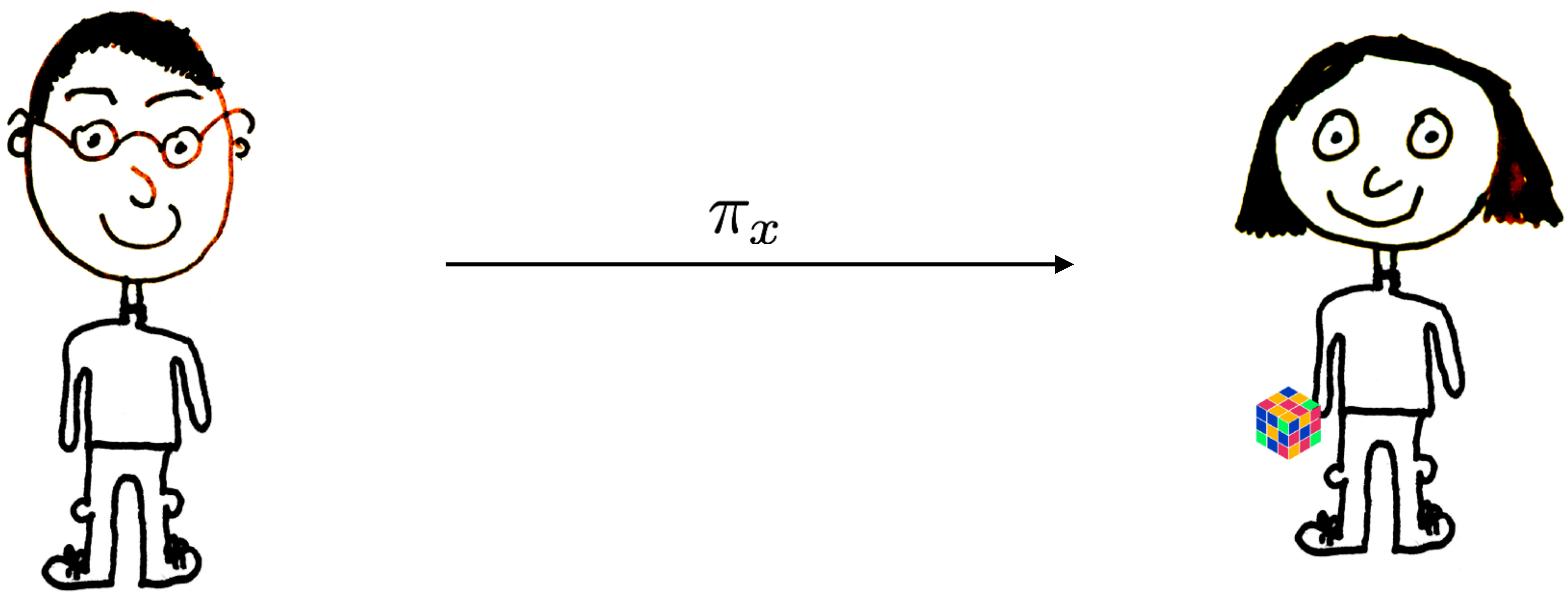

Prover $\cancel{k_P}$

Verifier $\cancel{k_V}$

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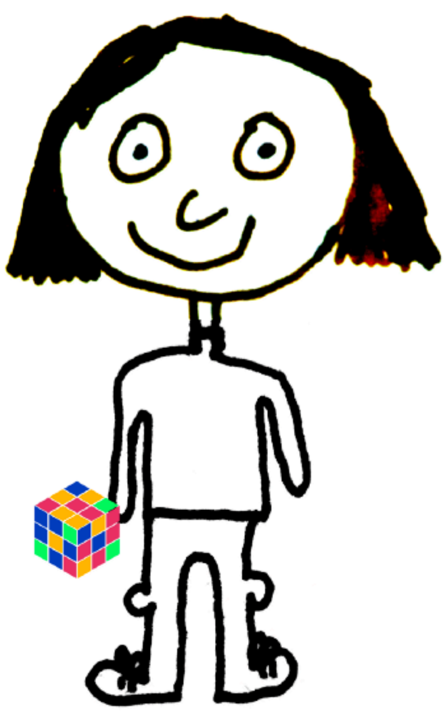
NP Language $\mathcal{L}$

Prover



Verifier

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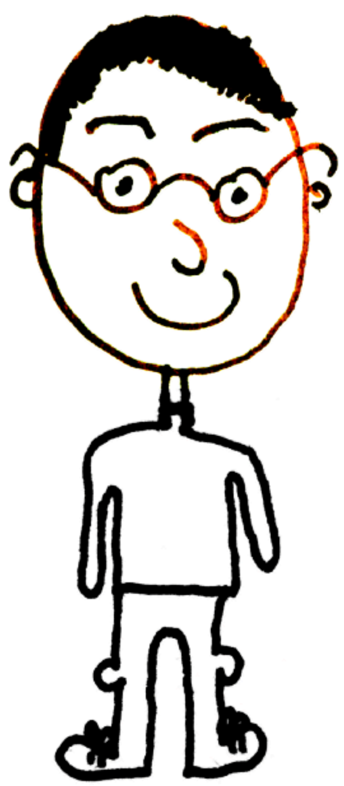
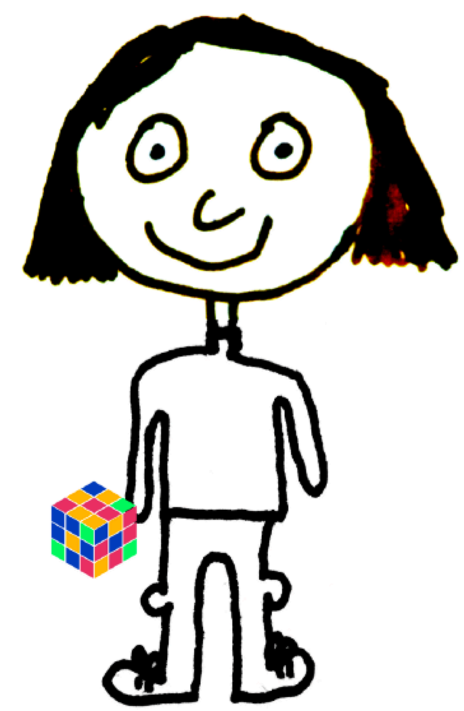


Prover



Verifier

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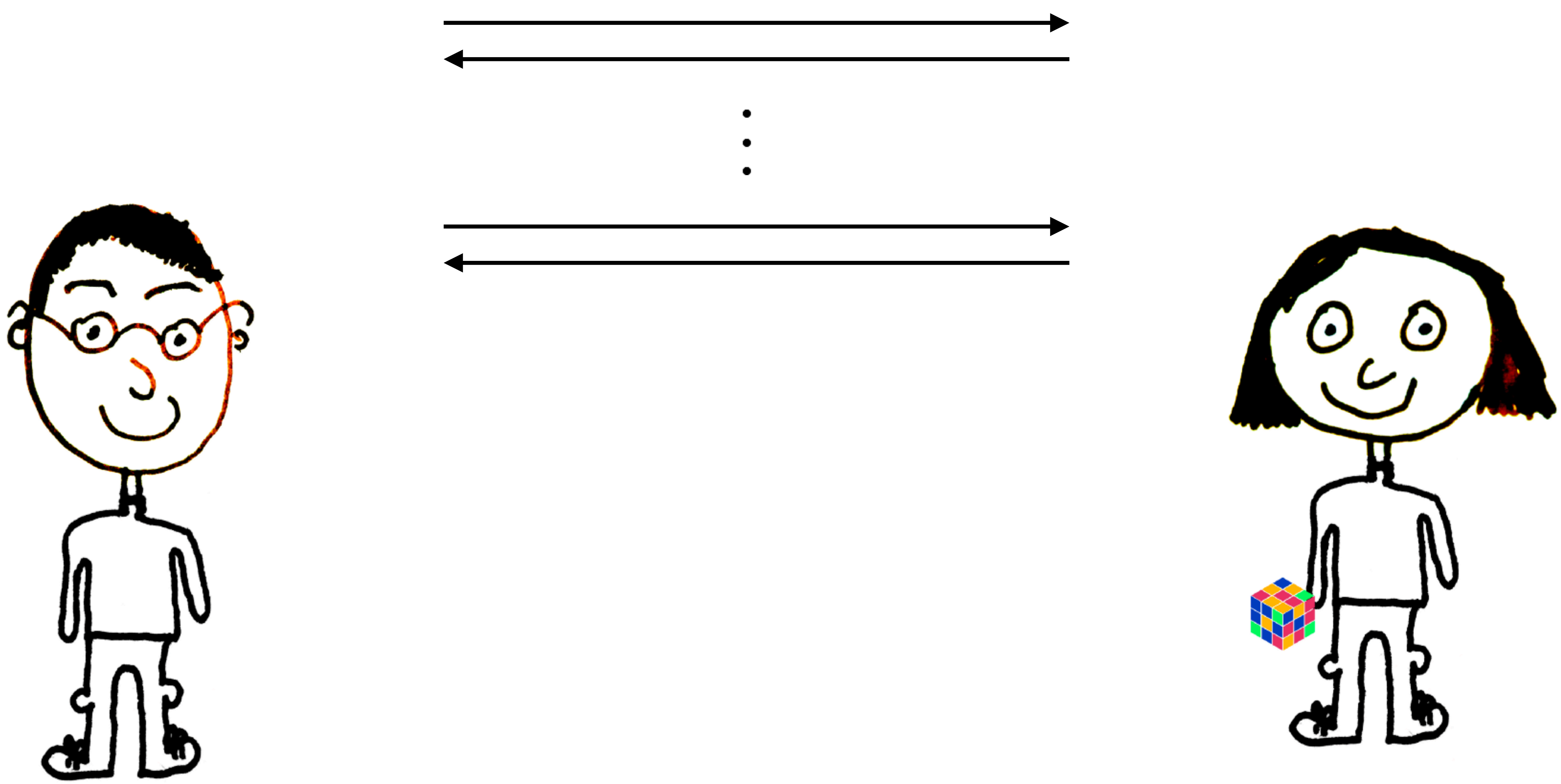


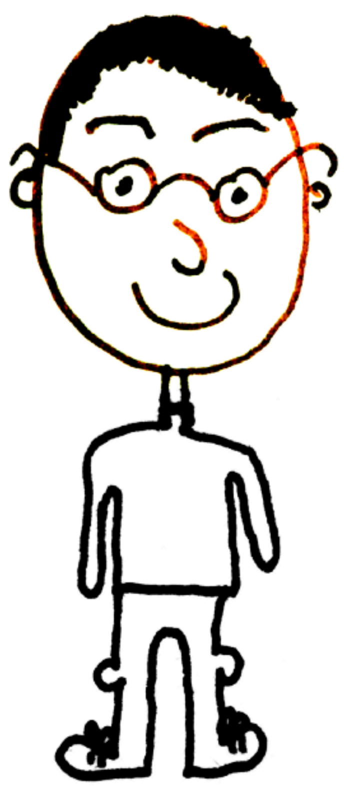
Prover

Verifier

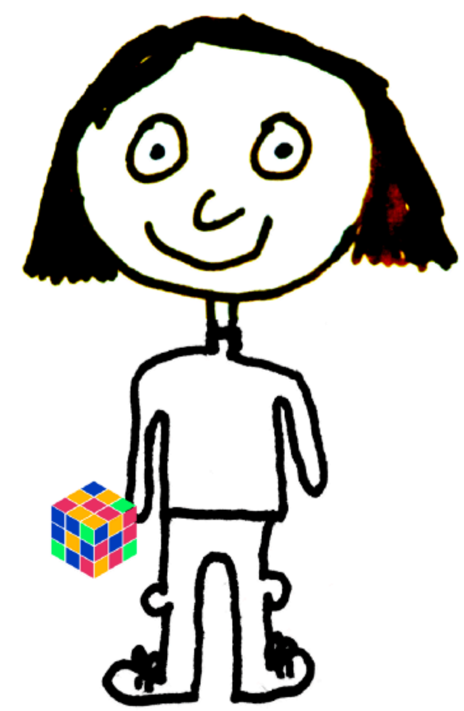
Easier to construct: follows from OWF [DMP88, …]

NIZK with Preprocessing [DMP88]
(Multi-Theorem)

NP Language $\mathcal{L}$



Prover
k_P



Verifier
k_V

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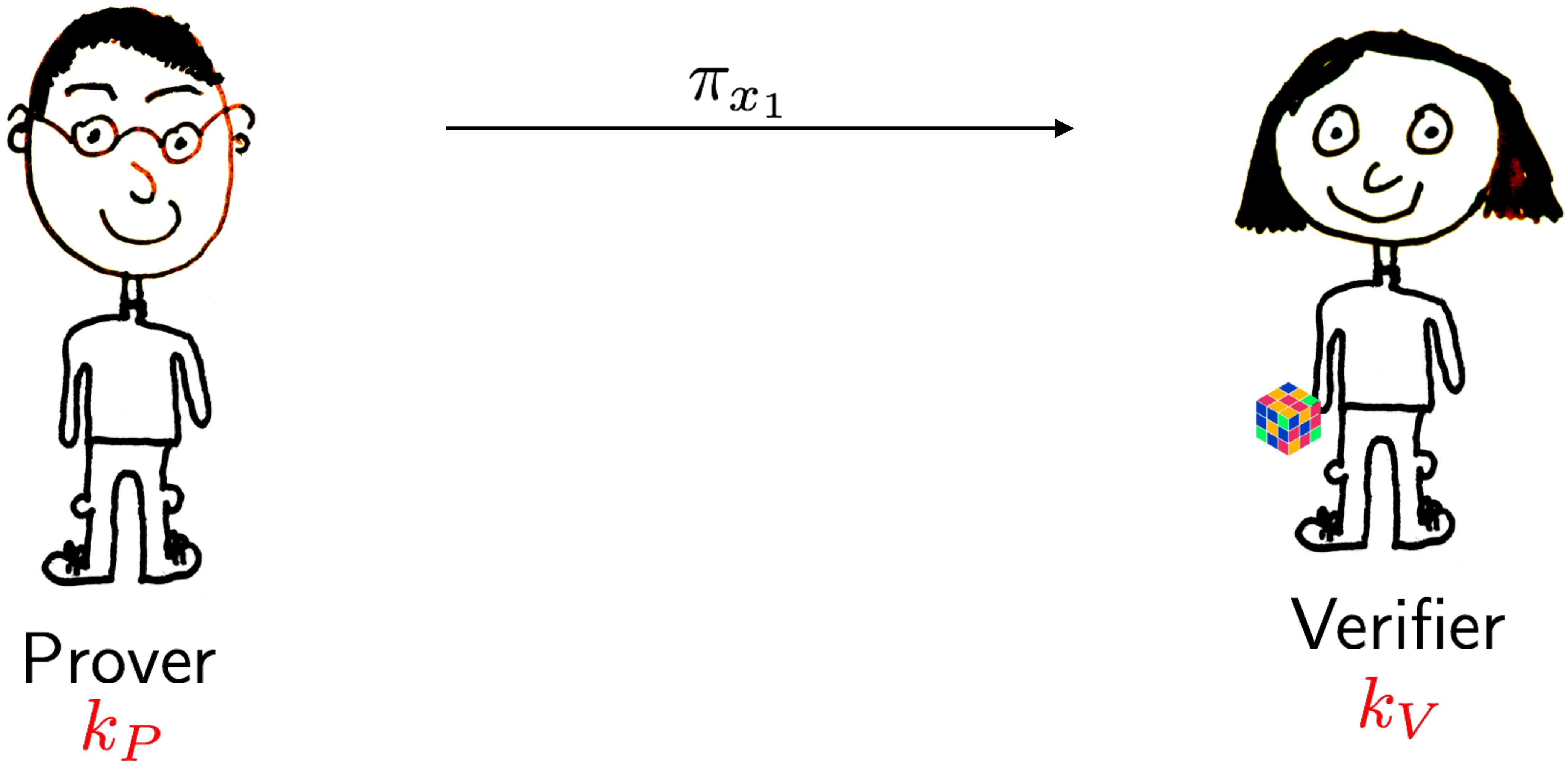

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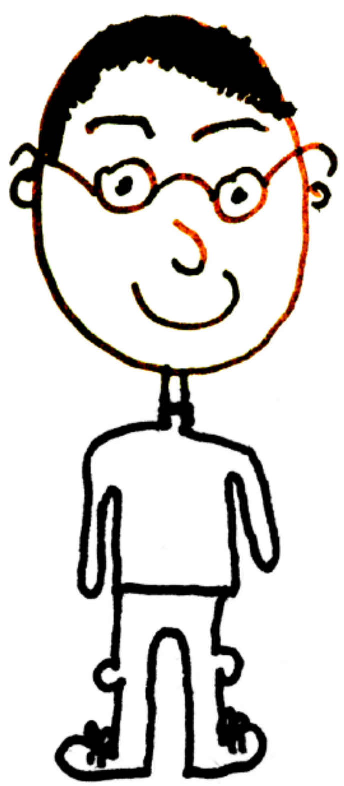
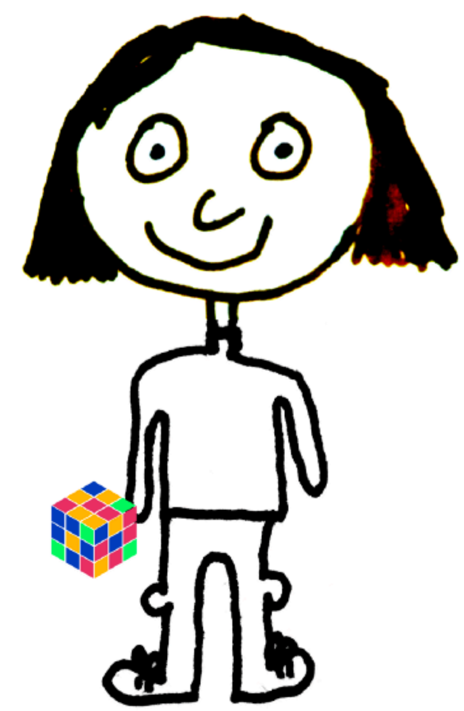


$\xrightarrow{\pi_{x_2}}$



Prover
k_P

Verifier
k_V

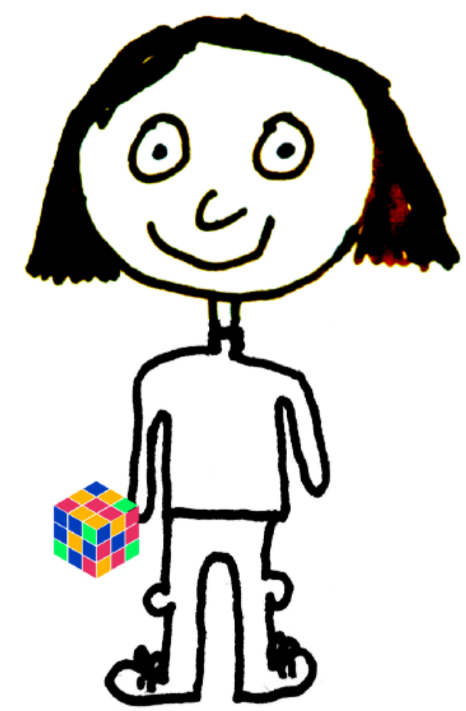
NIZK with Preprocessing [DMP88]
(Multi-Theorem)

NP Language $\mathcal{L}$

Prover

k_P

$$\xrightarrow{\pi_{x_3}}$$



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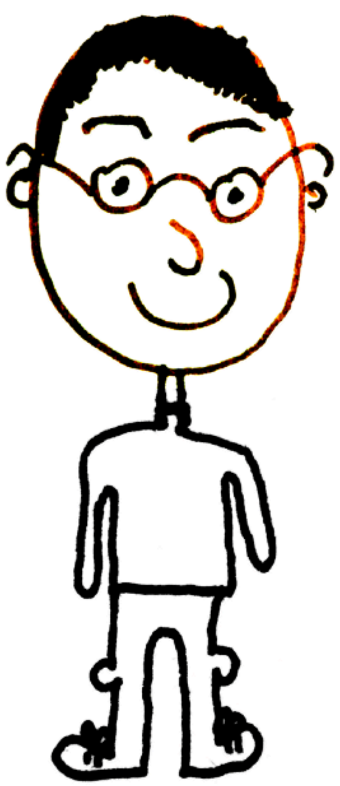
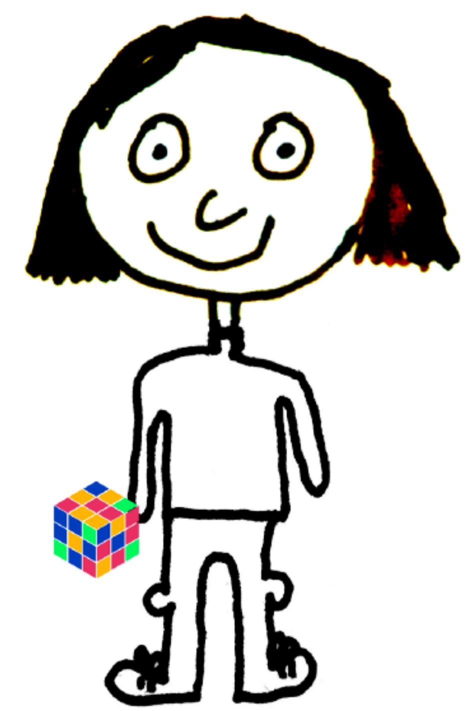


Prover
k_P

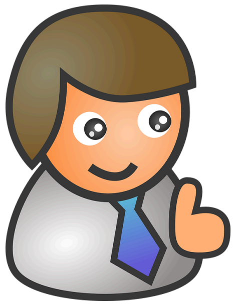
$$\xrightarrow{\pi_{x_3}}$$

$\vdots$



Verifier
k_V

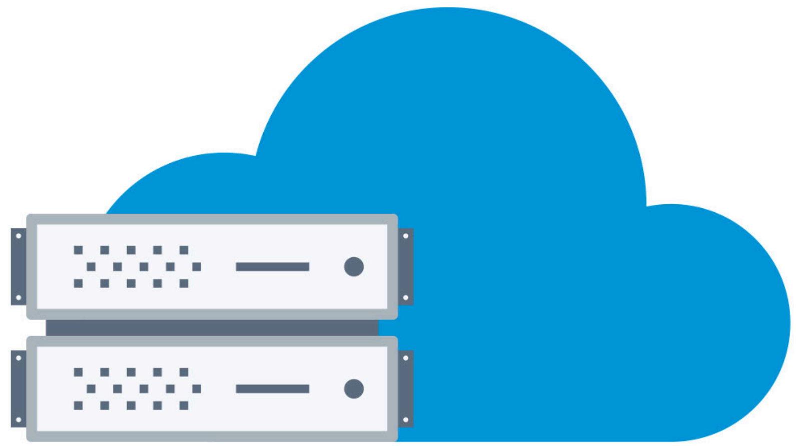
Homomorphic Signatures [BF11, GVW15]



$(\mathsf{sk}, \mathsf{pk})$

Homomorphic Signatures [BF11, GVW15]

(sk, pk)



$\mathbf{x} \in \{0,1\}^N, \quad \sigma_{\mathbf{x}}$

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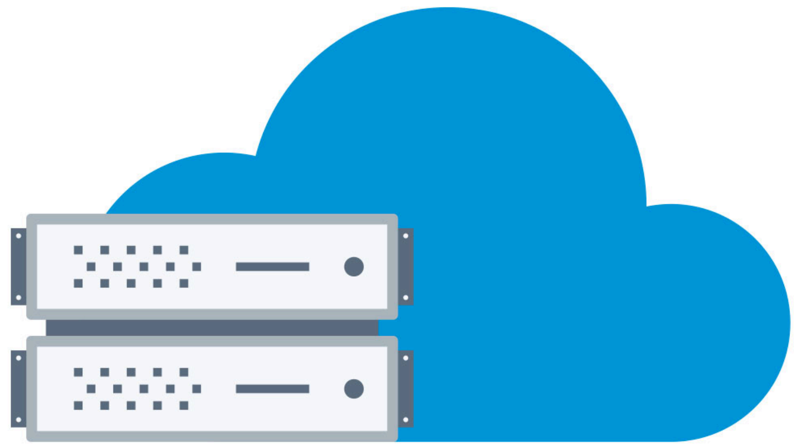
(sk, pk)



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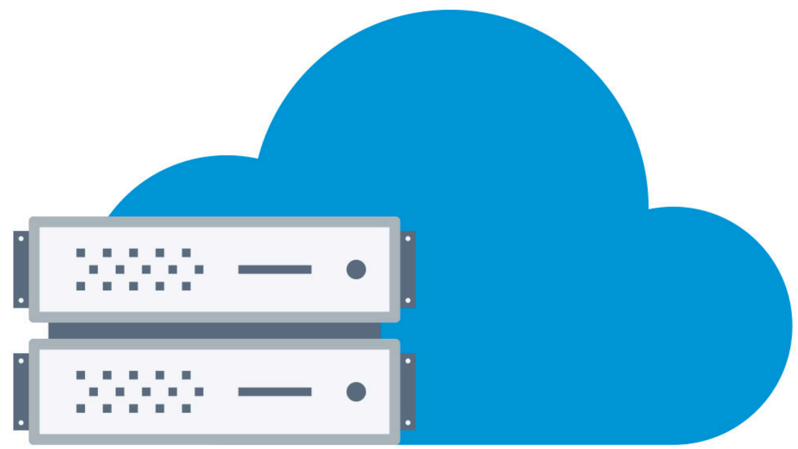
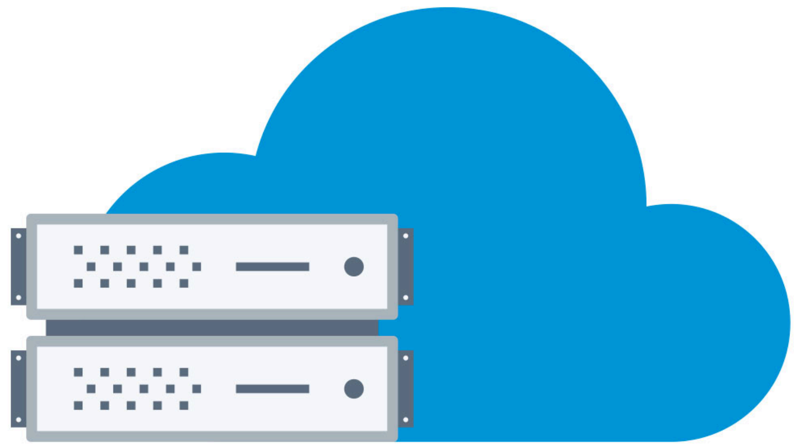


$\mathbf{x} \in \{0,1\}^N, \quad \sigma_{\mathbf{x}}$

$y = f(\mathbf{x}), \ \sigma^*_{\mathbf{x},f}$

Homomorphic Signatures [BF11, GVW15]

(sk, pk)



$\mathbf{x} \in \{0,1\}^N, \quad \sigma_{\mathbf{x}}$

$y = f(\mathbf{x}), \; \sigma^*_{\mathbf{x},f} \longrightarrow$

$\mathsf{Verify}_{\mathsf{pk}}(f, y, \sigma^*_{\mathbf{x},f})$

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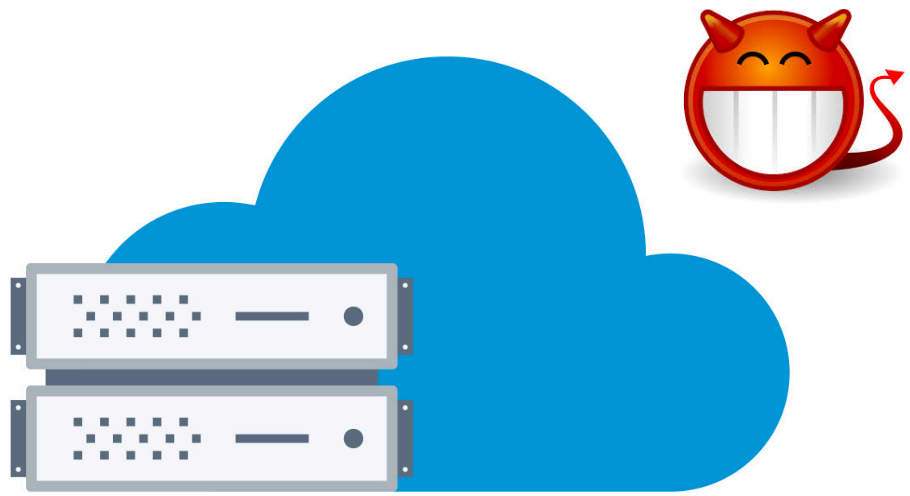

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Unforgeability?

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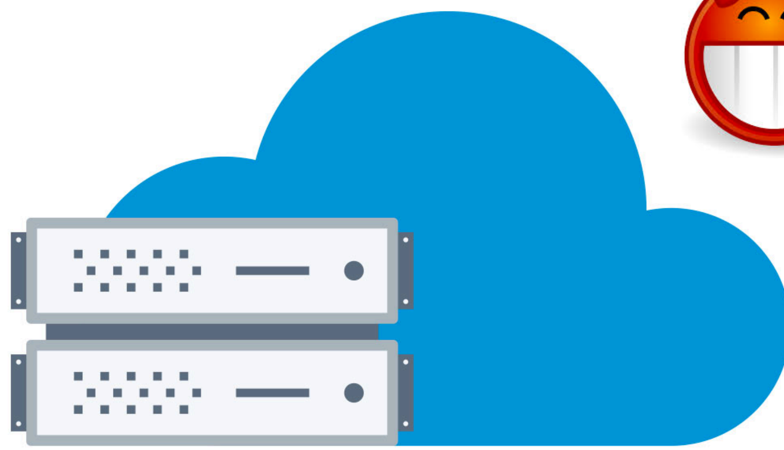

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Unforgeability?

$y' \neq f(\mathbf{x}), \ \sigma^*_{\mathbf{x},f}$

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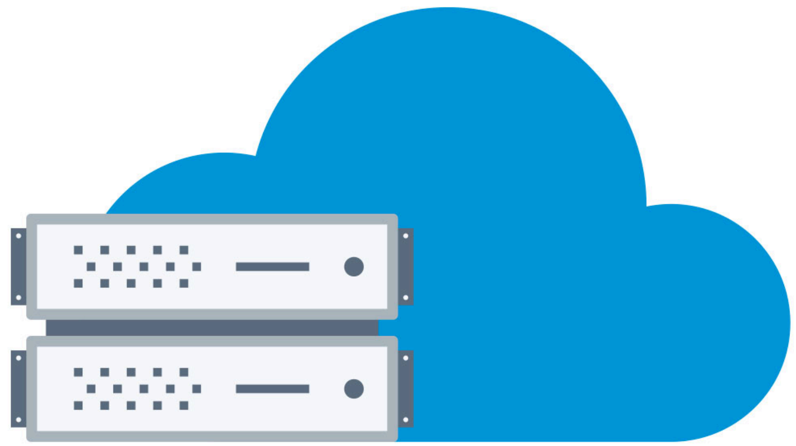


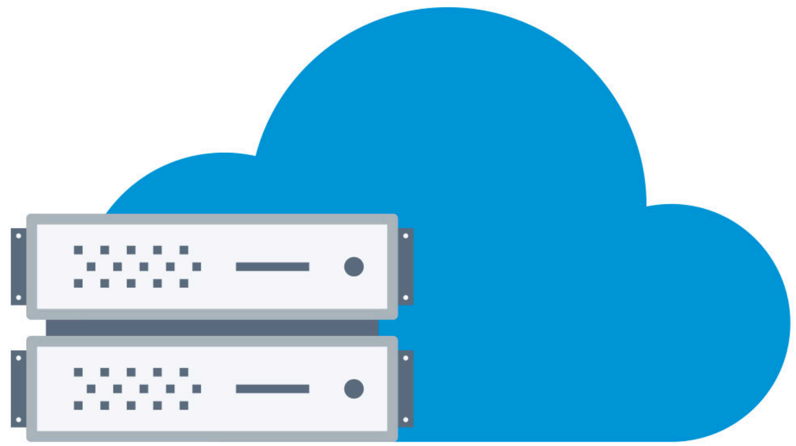
$\mathbf{x} \in \{0,1\}^N, \quad \sigma_{\mathbf{x}}$

$y = f(\mathbf{x}),\ \sigma^*_{\mathbf{x},f}$

Compactness: signature size $|\sigma^*_{\mathbf{x},f}|$ independent of $|\mathbf{x}|$

Homomorphic Signatures [BF11, GVW15]

(sk, pk)



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$y = f(\mathbf{x}),\ \sigma^*_{\mathbf{x},f} \longrightarrow$

$\mathsf{Verify}_{\mathsf{pk}}(f, y, \sigma^*_{\mathbf{x},f})$

Context-Hiding: $\sigma^*_{\mathbf{x},f}$ not reveal any more info about $\mathbf{x}$

Constructing Hom. Signatures [GVW15]

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Signature for $\mathbf{x} = (x_1, \ldots, x_N)$ consists of short matrices

$\sigma_{\mathbf{x}} = \mathbf{R}_1, \ldots, \mathbf{R}_N \in \mathbb{Z}^{m \times m}$ such that

$$\mathbf{A} \cdot \mathbf{R}_1 + x_1 \cdot \mathbf{G} = \mathbf{C}_1$$

$$\vdots$$

$$\mathbf{A} \cdot \mathbf{R}_N + x_N \cdot \mathbf{G} = \mathbf{C}_N$$

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$$\vdots$$

$$\mathbf{A} \cdot \mathbf{R}_N + x_N \cdot \mathbf{G} = \mathbf{C}_N$$

$$\xrightarrow{\text{GSW}} \quad \mathbf{A} \cdot \mathbf{R}_f + f(\mathbf{x}) \cdot \mathbf{G} = \mathbf{C}_f$$

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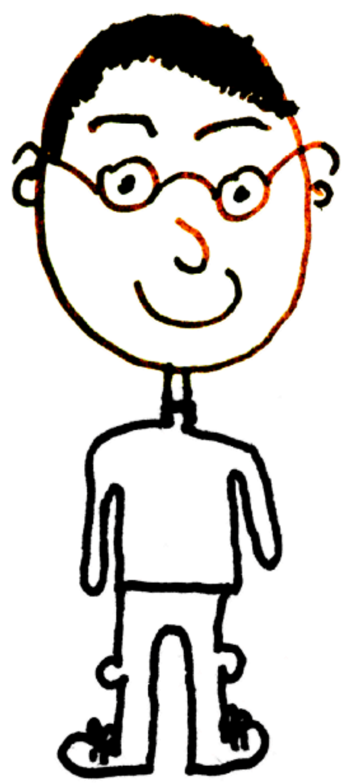
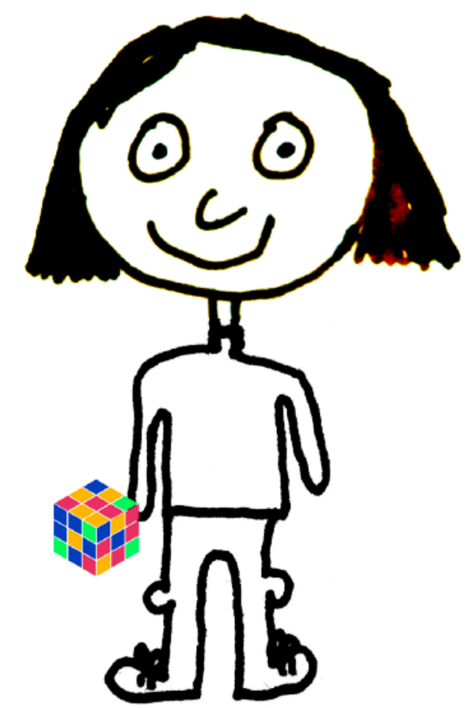
$$\begin{matrix} \mathbf{A} \cdot \mathbf{R}_1 + x_1 \cdot \mathbf{G} = \mathbf{C}_1 \\ \vdots \\ \mathbf{A} \cdot \mathbf{R}_N + x_N \cdot \mathbf{G} = \mathbf{C}_N \end{matrix} \xrightarrow{\text{GSW}} \mathbf{A} \cdot \mathbf{R}_f + f(\mathbf{x}) \cdot \mathbf{G} = \mathbf{C}_f$$

Need extra step for context-hiding

Homomorphic Signatures to NIZK?

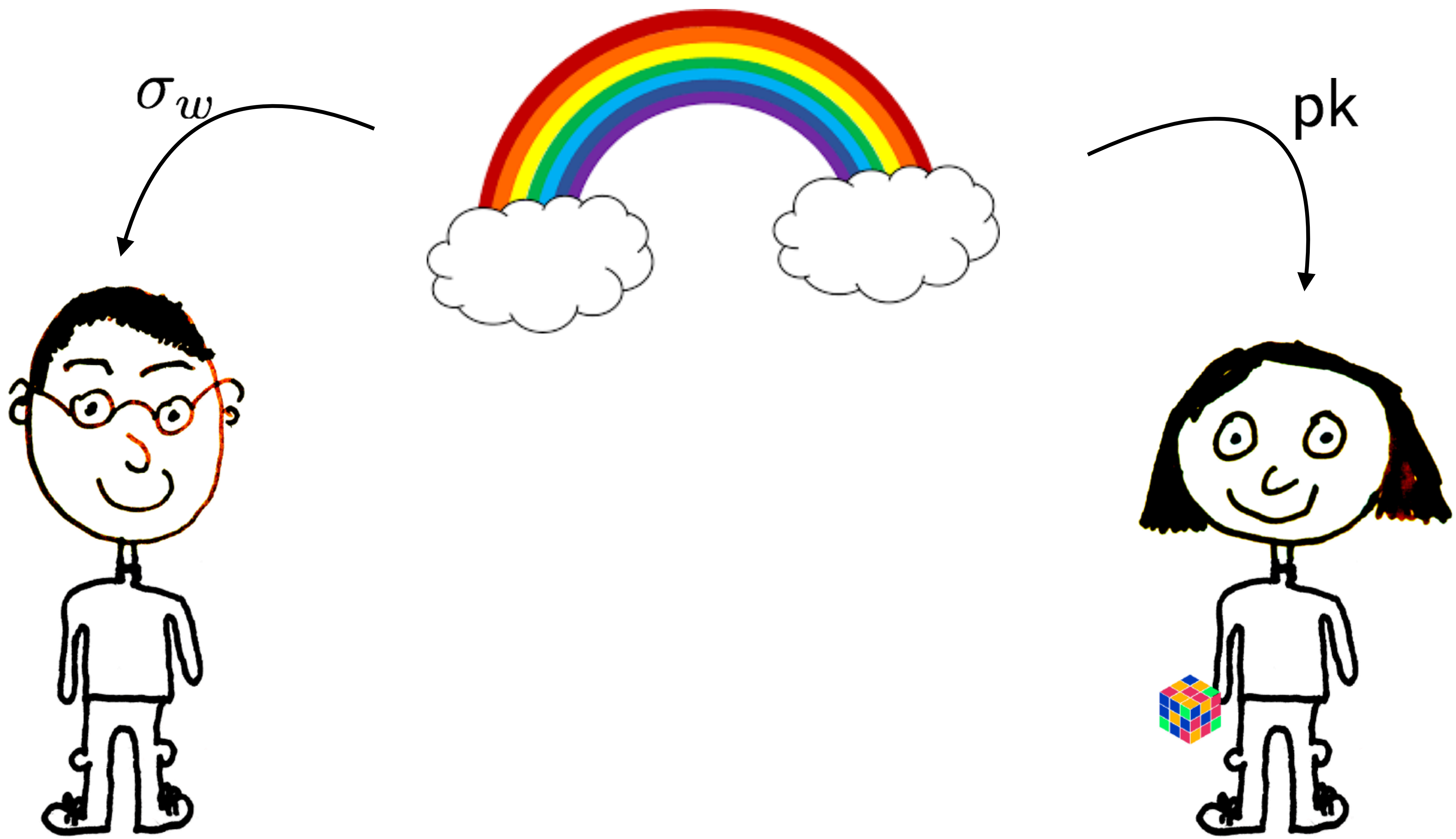


Prover(x, w)



Verifier(x)

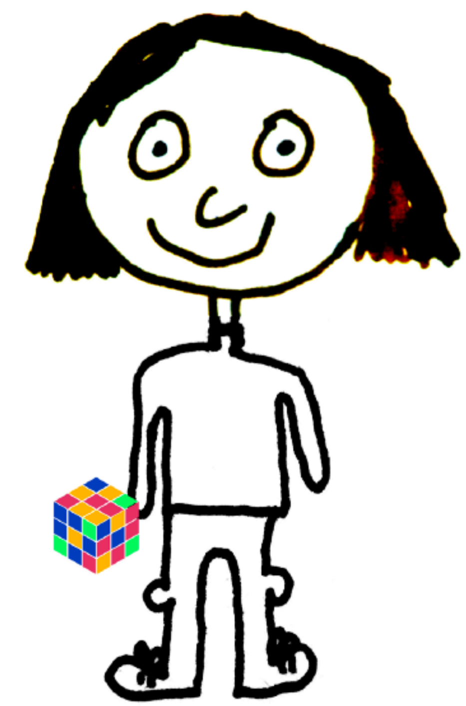
Homomorphic Signatures to NIZK?



$\mathsf{Prover}(x, w)$ $\mathsf{Verifier}(x)$

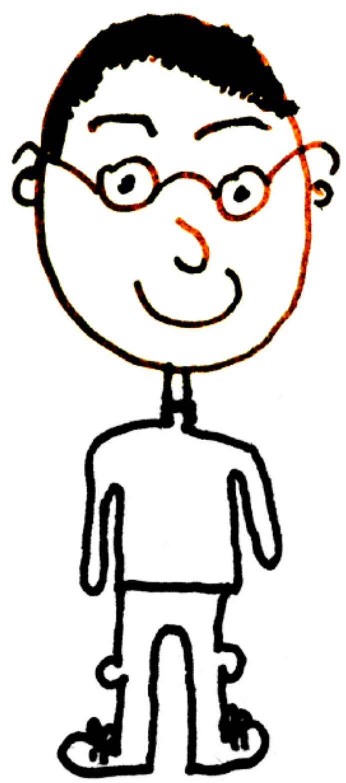
Homomorphic Signatures to NIZK?

$\mathsf{Prover}(x, w)$
σ_w

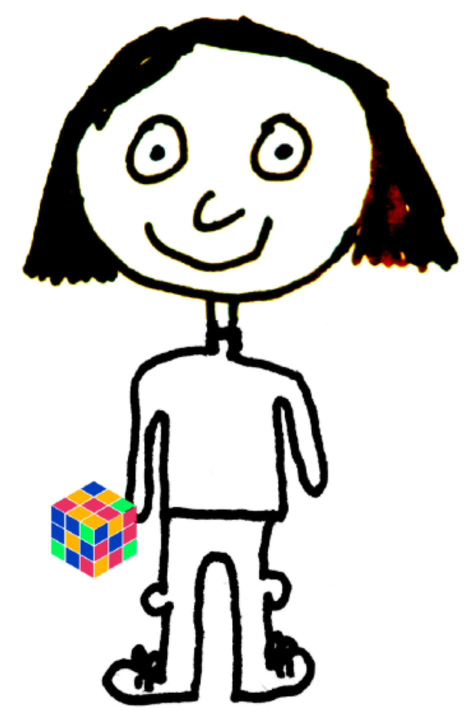


$\mathsf{Verifier}(x)$
pk

Homomorphic Signatures to NIZK?



$$\xrightarrow{\quad 1 = f_x(w),\ \sigma^*_{w,f_x} \quad}$$

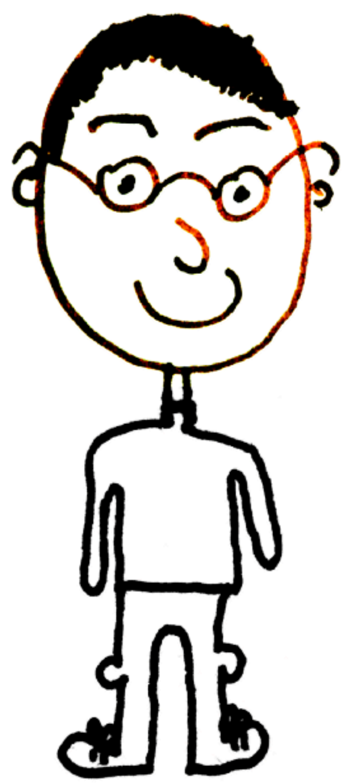


$\mathsf{Prover}(x, w)$
σ_w

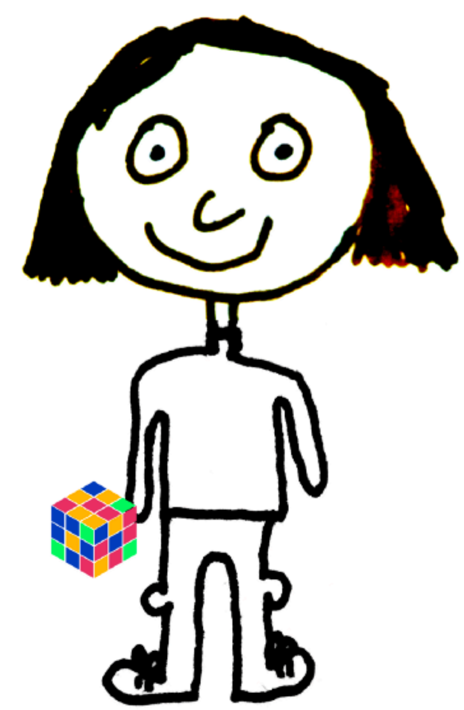
$\mathsf{Verifier}(x)$
pk

Homomorphic Signatures to NIZK?

$$f_x(w) = \mathcal{R}(x, w)$$



$$\xrightarrow{1 = f_x(w),\ \sigma^*_{w,f_x}}$$



Prover(x, w)
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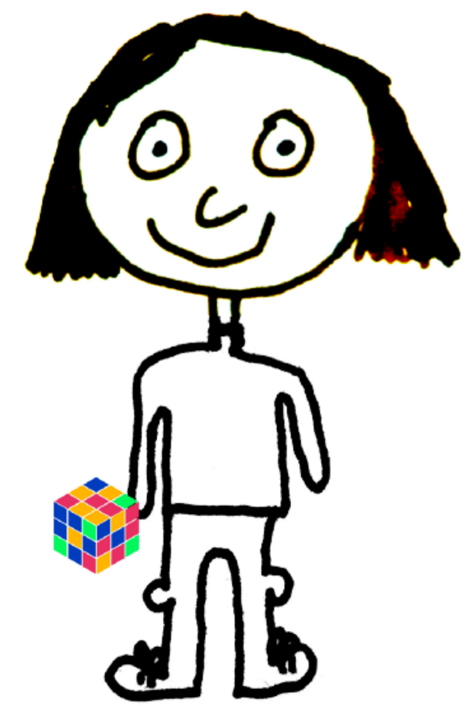
Homomorphic Signatures to NIZK?

$$f_x(w) = \mathcal{R}(x, w)$$

$$1 = f_x(w),\ \sigma^*_{w,f_x} \longrightarrow$$



$\mathsf{Prover}(x, w)$

σ_w

$\mathsf{Verifier}(x)$

$\mathsf{Verify}_{\mathsf{pk}}(f_x, 1, \sigma^*_{w,f_x})$

Homomorphic Signatures to NIZK?

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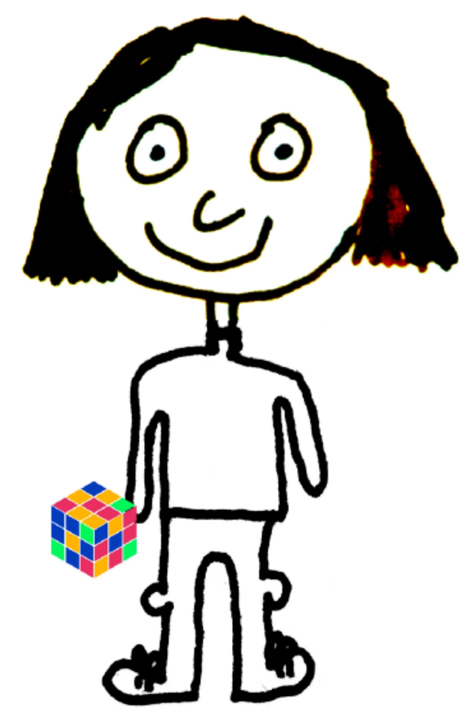
$$\xrightarrow{1 = f_x(w),\ \sigma^*_{w,f_x}}$$



$\mathsf{Prover}(x, w)$

σ_w

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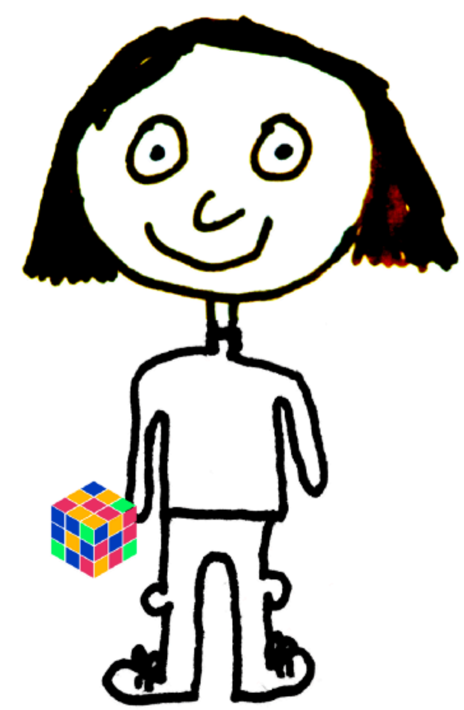
$\mathsf{Verify}_{\mathsf{pk}}(f_x, 1, \sigma^*_{w,f_x})$

1. **HS Correctness** implies **NIZK Completeness**

Homomorphic Signatures to NIZK?

$$f_x(w) = \mathcal{R}(x, w)$$

$$\xrightarrow{\quad 1 \neq f_x(w),\ \ \sigma^*_{w,f_x} \quad}$$



$\mathsf{Prover}(x, w)$

σ_w

$\mathsf{Verifier}(x)$

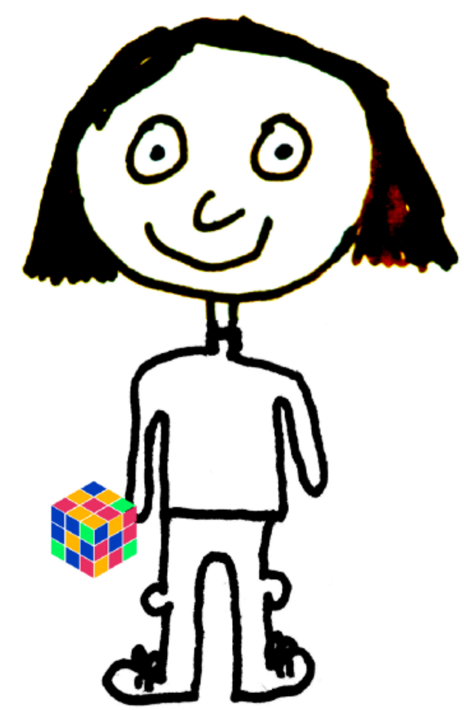
$\mathsf{Verify}_{\mathsf{pk}}(f_x, 1, \sigma^*_{w,f_x})$

2. **HS Unforgeability** implies **NIZK Soundness**

Homomorphic Signatures to NIZK?

$$f_x(w) = \mathcal{R}(x, w)$$

$$\xrightarrow{\quad 1 = f_x(w),\ \sigma^*_{w,f_x} \quad}$$



Prover(x, w)

σ_w

Verifier(x)

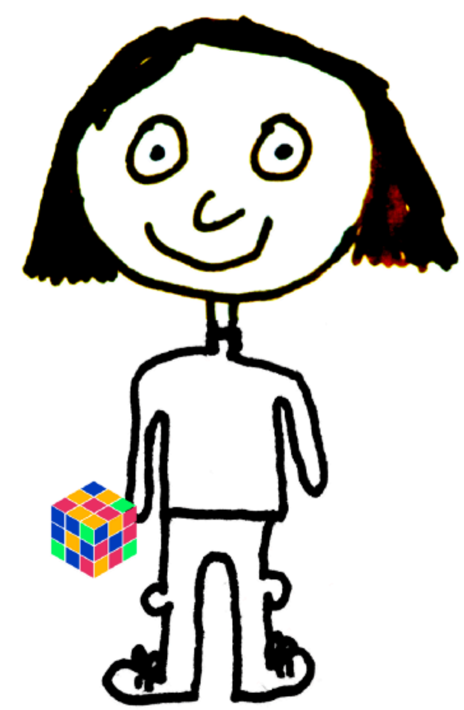
Verify$_{\mathsf{pk}}(f_x, 1, \sigma^*_{w,f_x})$

3. **HS Context-Hiding** implies **NIZK Zero-Knowledge**

Homomorphic Signatures to NIZK?

$$f_x(w) = \mathcal{R}(x, w)$$

$$1 = f_x(w),\ \sigma^*_{w,f_x}$$



$\mathsf{Prover}(x, w)$

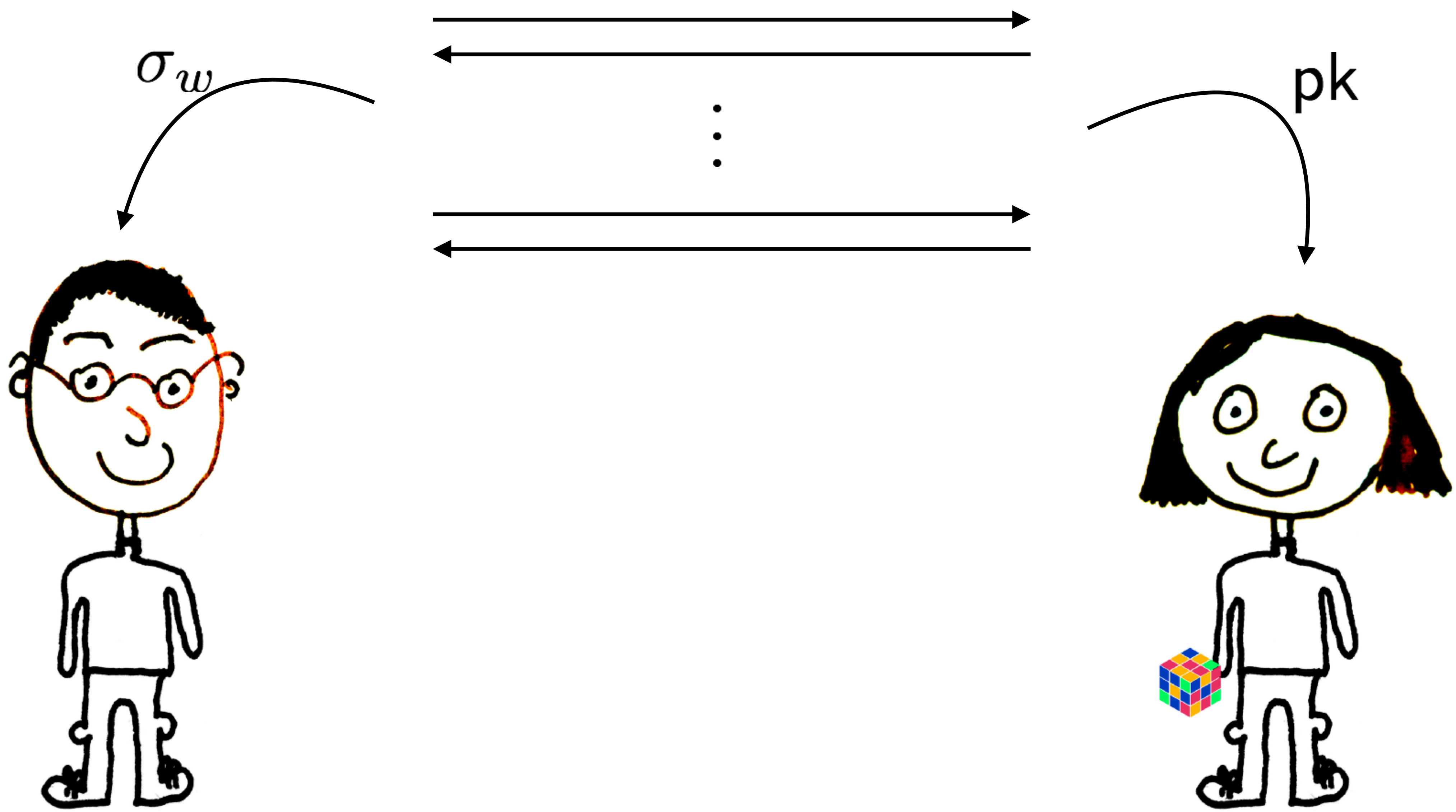
σ_w

$\mathsf{Verifier}(x)$

$\mathsf{Verify}_{\mathsf{pk}}(f_x, 1, \sigma^*_{w,f_x})$

How can the prover get its witness signed?

Homomorphic Signatures to NIZK?



Prover(x, w) Verifier(x)

σ_w pk

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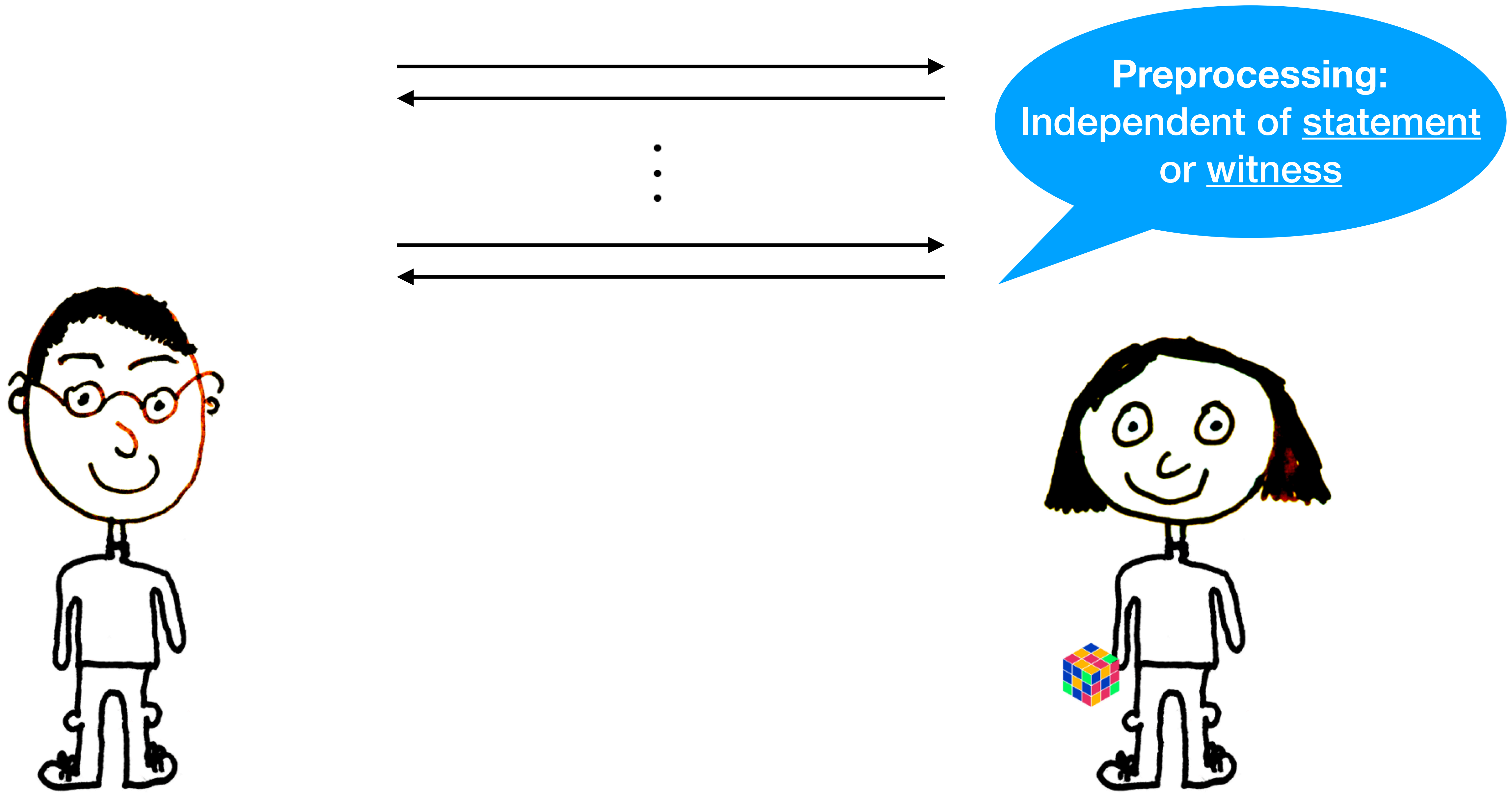

Prover(x, w)
σ_w

Verifier(x)
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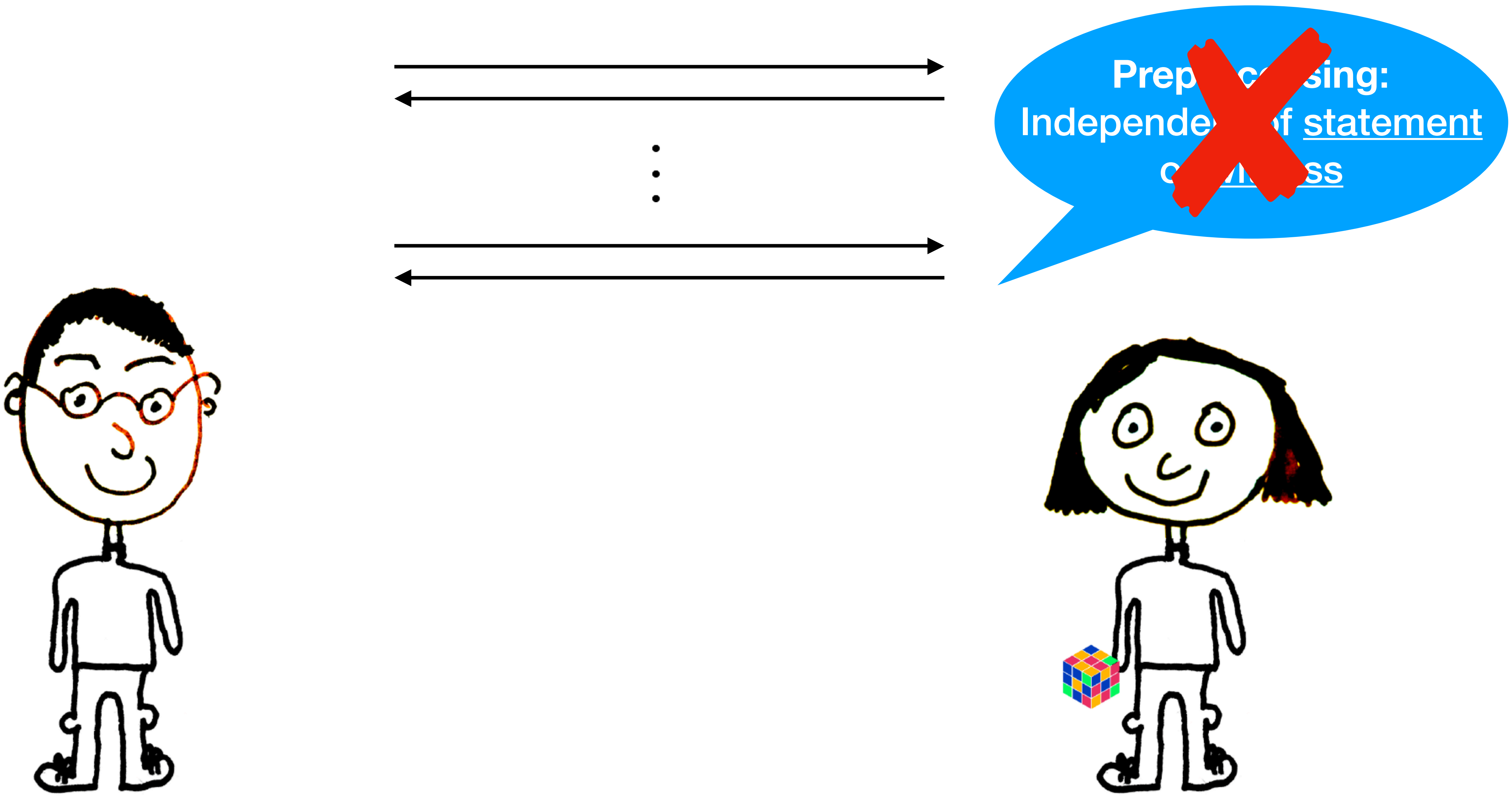


Prover(x, w)
σ_w

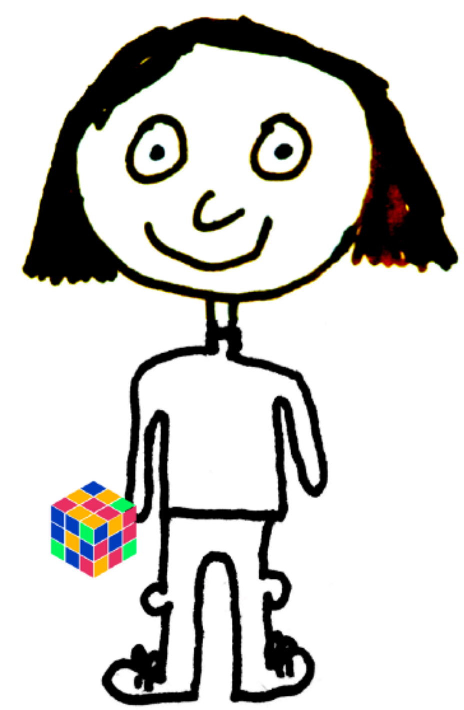
Verifier(x)
pk

How can the prover get its witness signed?

Homomorphic Signatures to NIZK?

$$\mathsf{Enc} : \mathcal{K} \times \mathcal{M} \to \mathcal{C}$$
$$\mathsf{Dec} : \mathcal{K} \times \mathcal{C} \to \mathcal{M}$$

$\mathsf{Prover}(x, w)$
σ_k



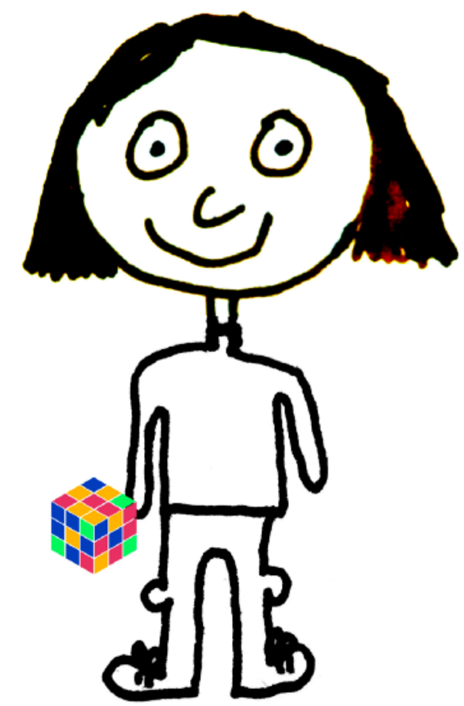
$\mathsf{Verifier}(x)$
pk

Homomorphic Signatures to NIZK?

$$\mathsf{Enc} : \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{C}$$
$$\mathsf{Dec} : \mathcal{K} \times \mathcal{C} \rightarrow \mathcal{M}$$

$$\xrightarrow{\mathsf{ct} = \mathsf{Enc}(k, w)}$$

$$\xrightarrow{\sigma^*_{k, g_x, \mathsf{ct}}}$$



$\mathsf{Prover}(x, w)$
σ_k

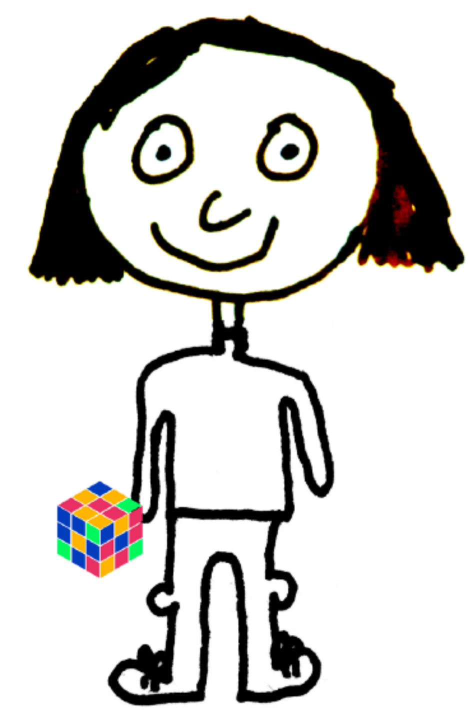
$\mathsf{Verifier}(x)$
pk

Homomorphic Signatures to NIZK?

$$g_{x,\mathsf{ct}}(k) = \mathcal{R}(x, \mathsf{Dec}(k, \mathsf{ct}))$$

$\mathsf{ct} = \mathsf{Enc}(k, w)$

$\sigma^*_{k, g_{x,\mathsf{ct}}}$



$\mathsf{Prover}(x, w)$
σ_k

$\mathsf{Verifier}(x)$
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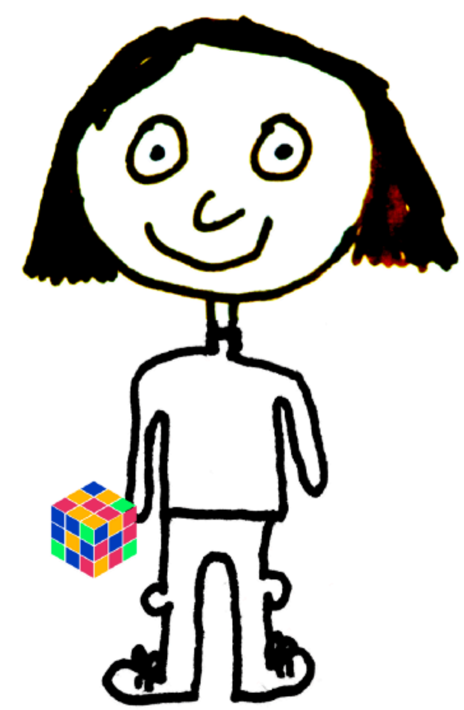
Homomorphic Signatures to NIZK?

$$g_{x,\mathsf{ct}}(k) = \mathcal{R}(x, \mathsf{Dec}(k, \mathsf{ct}))$$

$$\xrightarrow{\mathsf{ct} = \mathsf{Enc}(k, w)}$$

$$\xrightarrow{\sigma^*_{k, g_{x,\mathsf{ct}}}}$$



$\mathsf{Prover}(x, w)$

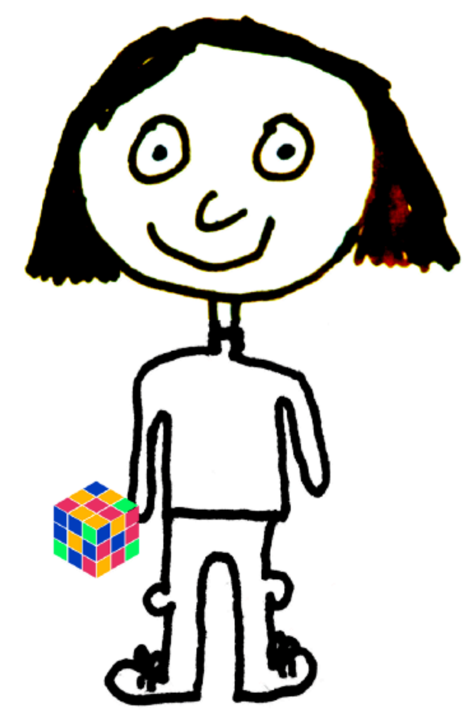
σ_k

$\mathsf{Verifier}(x)$

$\mathsf{Verify}_{\mathsf{pk}}(g_{x,\mathsf{ct}}, 1, \sigma^*_{k, g_{x,\mathsf{ct}}})$

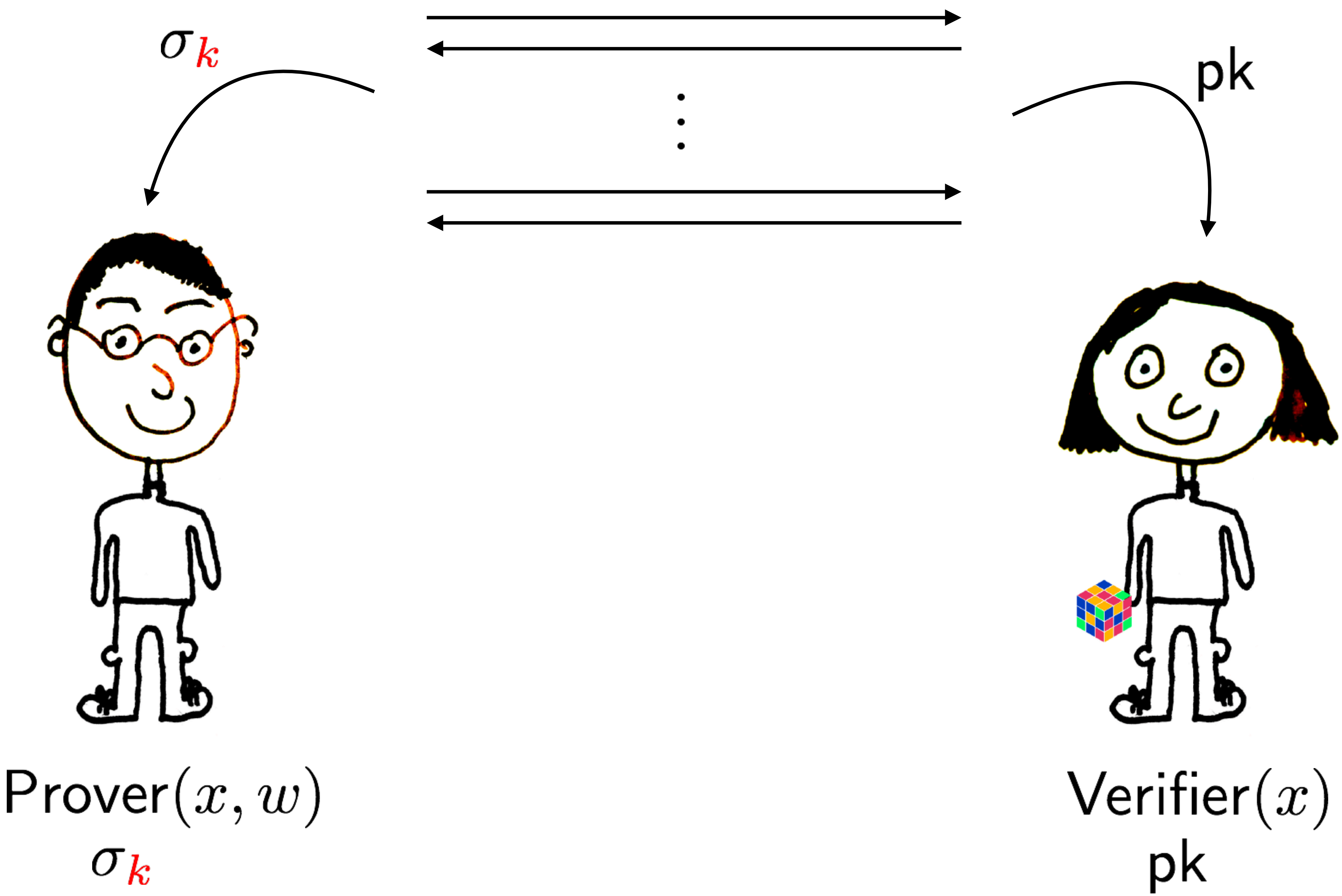
Homomorphic Signatures to NIZK?

Prover(x, w)
σ_k

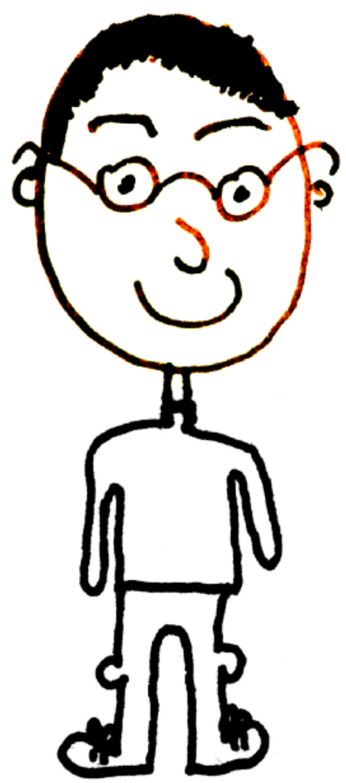


Verifier(x)
pk

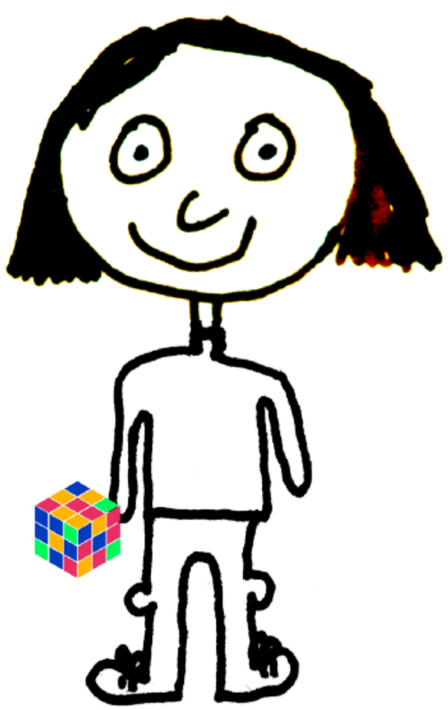
Homomorphic Signatures to NIZK?



Homomorphic Signatures to NIZK?



Prover(x, w)
σ_k

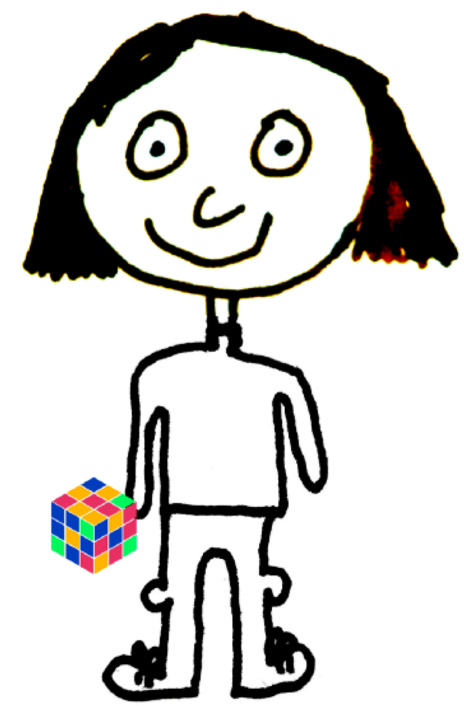


Verifier(x)
pk

Homomorphic Signatures to NIZK?

Prover(x, w)
σ_{k}

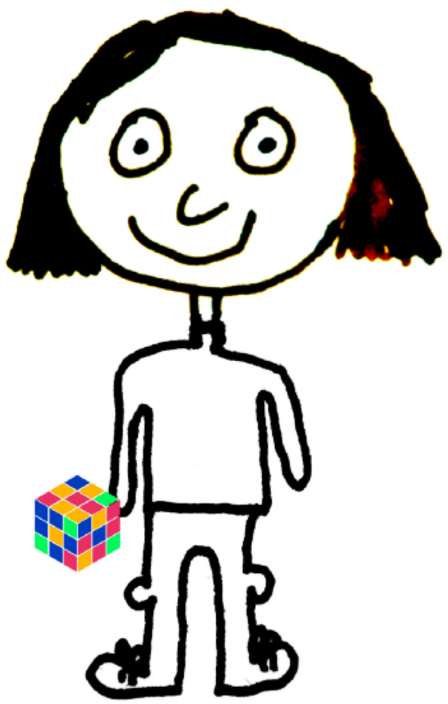
$$\pi_1 = (\mathsf{ct}_1, \sigma_1^*) \longrightarrow$$



Verifier(x)
pk

Homomorphic Signatures to NIZK?

$$\pi_2 = (\mathsf{ct}_2, \sigma_2^*) \longrightarrow$$



$\mathsf{Prover}(x, w)$
σ_{k}

$\mathsf{Verifier}(x)$
pk

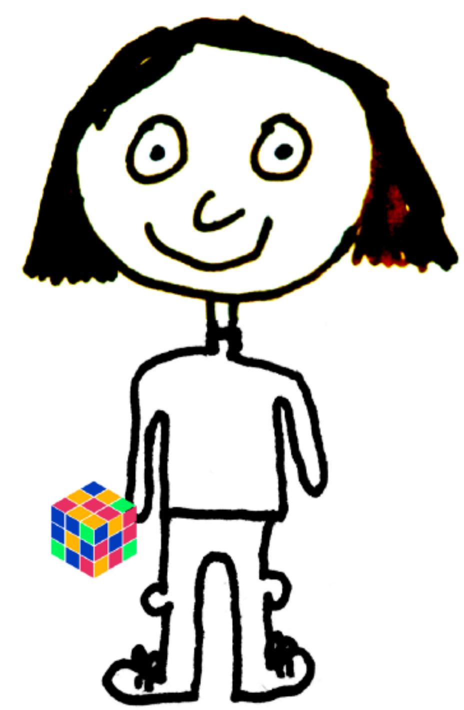
Homomorphic Signatures to NIZK?

$\mathsf{Prover}(x, w)$

σ_{k}

$$\xrightarrow{\pi_3 = (\mathsf{ct}_3, \sigma_3^*)}$$

$$\vdots$$



$\mathsf{Verifier}(x)$

pk

How should we do <u>Preprocessing</u>?

Generically: Either have to use many rounds or non-black use (costly)

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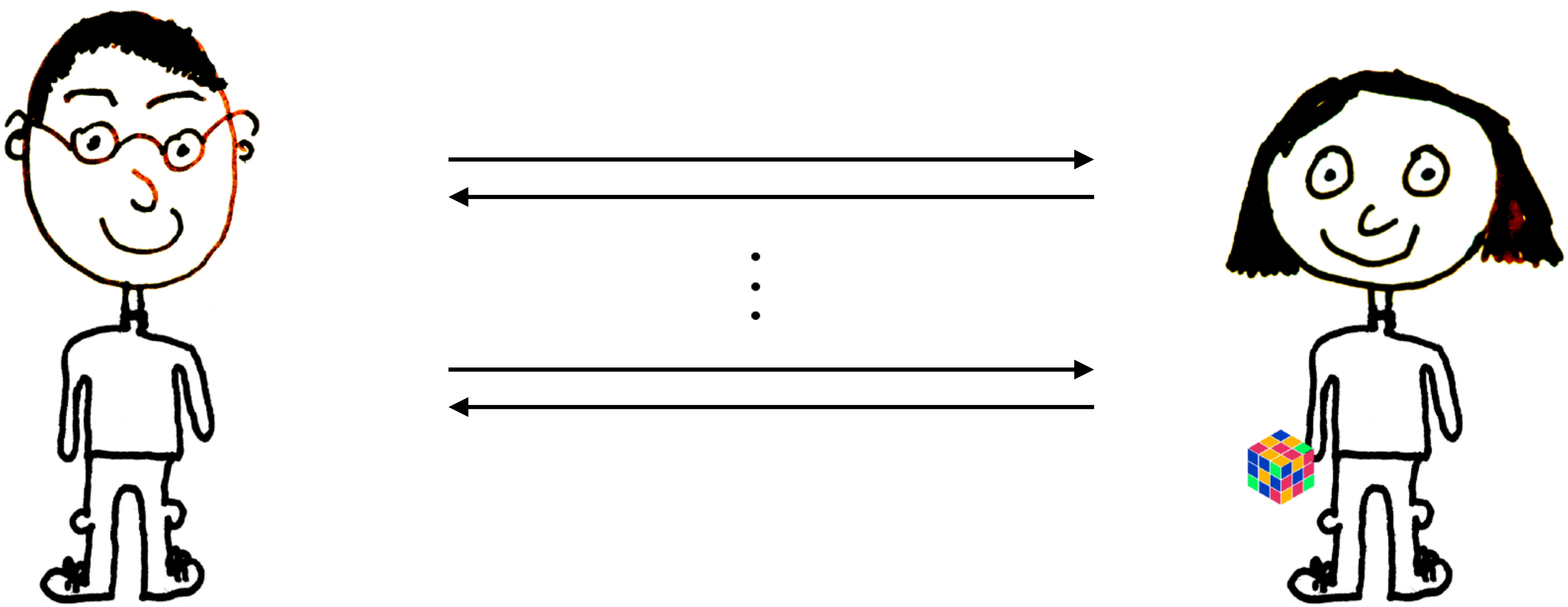
Let's just construct directly!

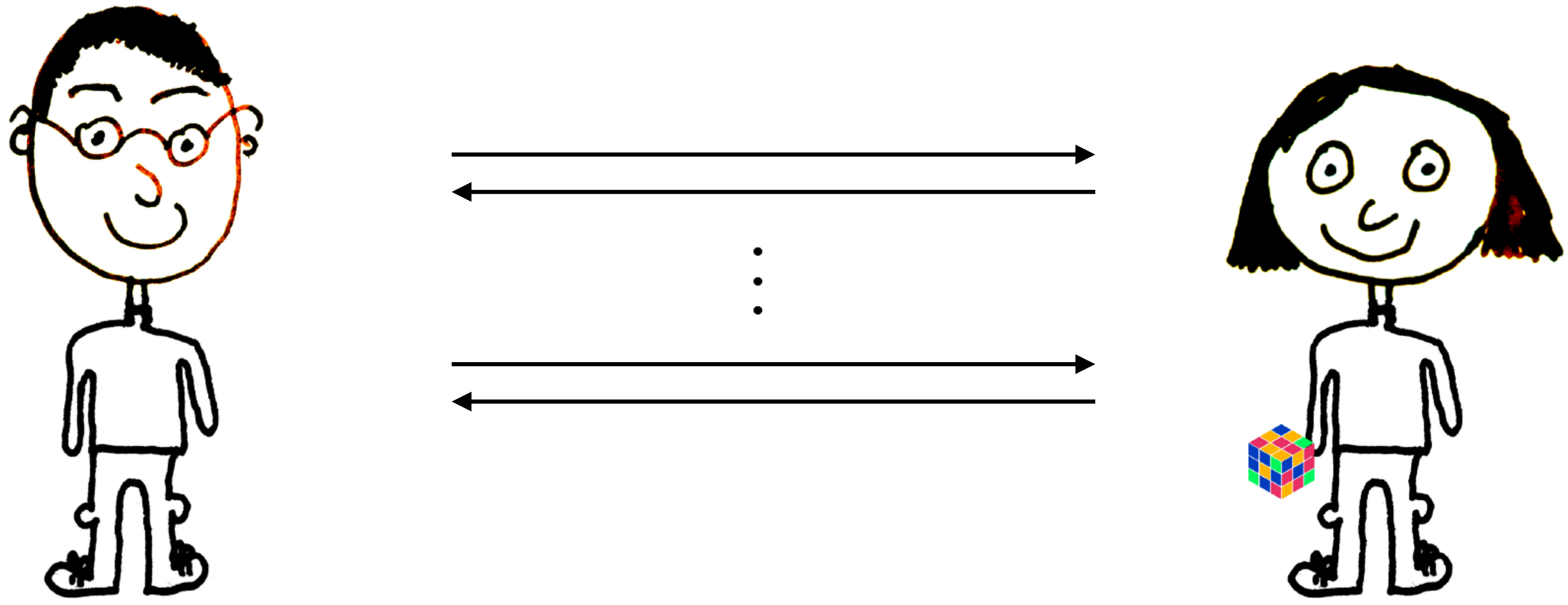
How should we do **Preprocessing**?



Prover(x, w)
σ_{k}

Verifier(x)
pk

How should we do **Preprocessing**?



Receiver

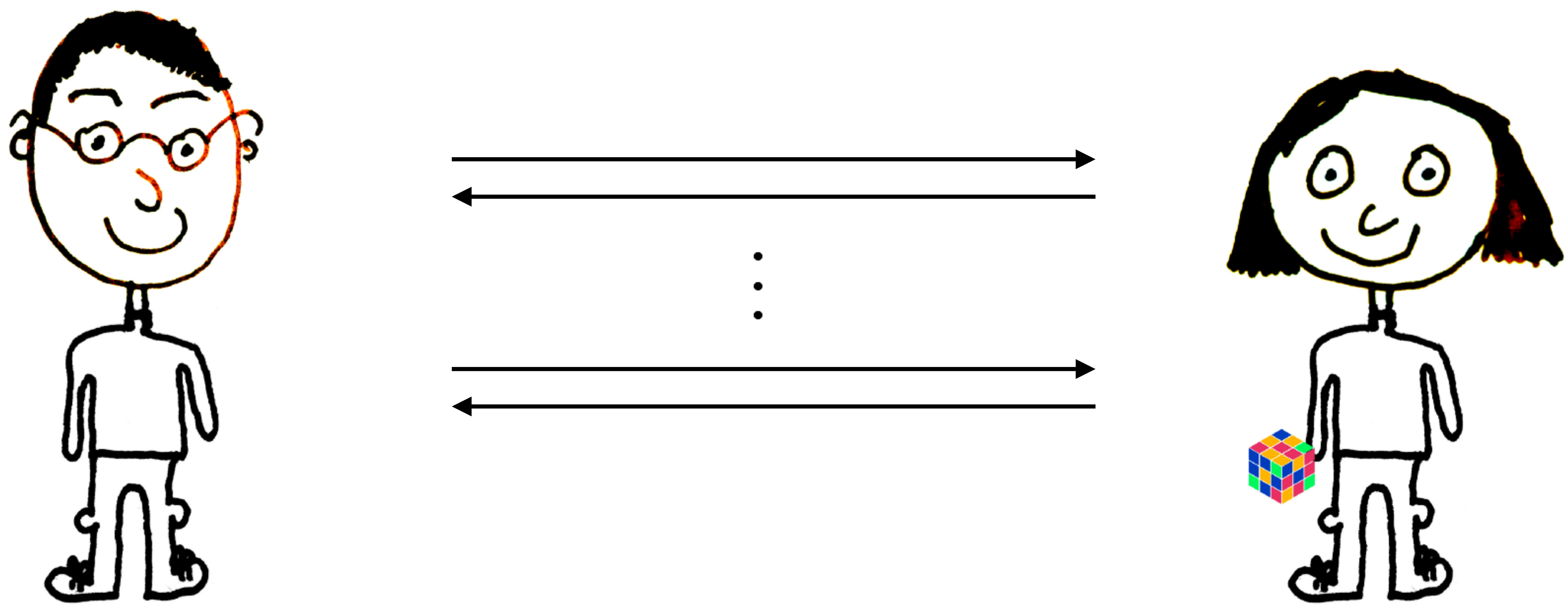
σ_m

Signer

$(\mathsf{sk}, \mathsf{pk})$

How should we do **Preprocessing**?

Blind Signatures?



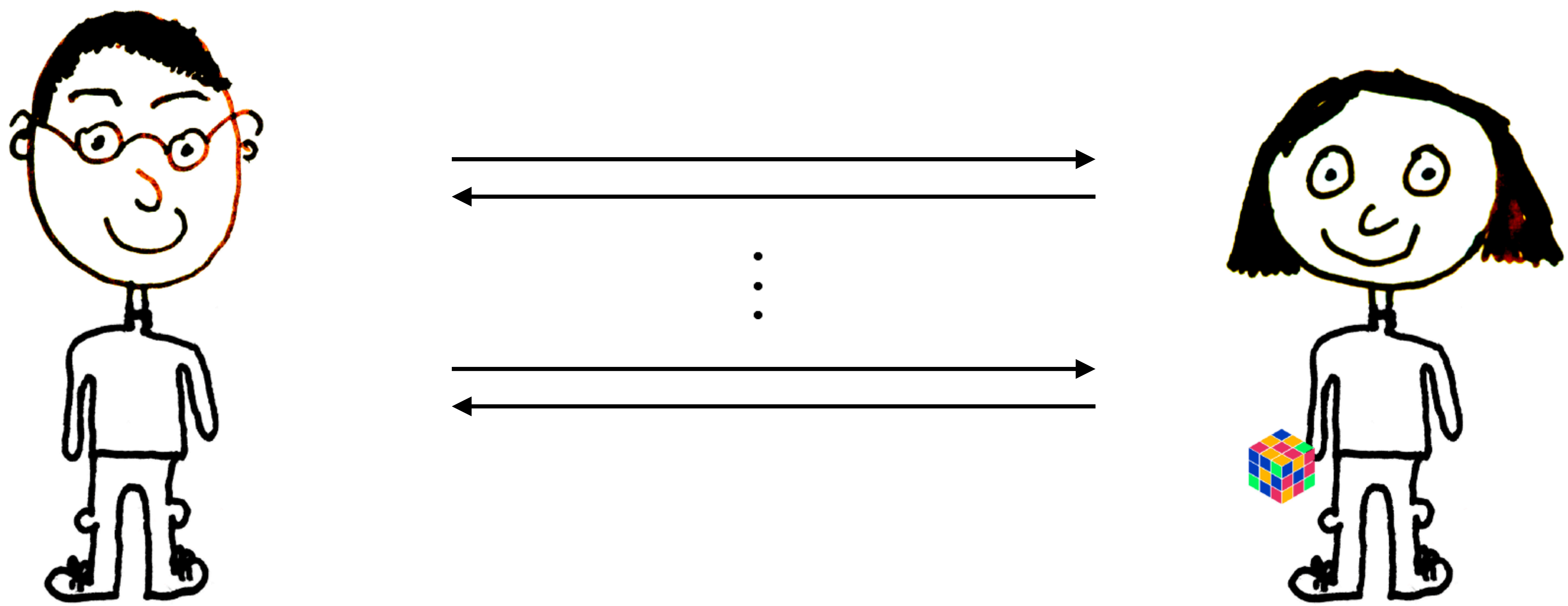
Receiver σ_m

Signer $(\mathsf{sk}, \mathsf{pk})$

How should we do **Preprocessing**?

Blind Signatures?

Blind Homomorphic Signatures (BHS)



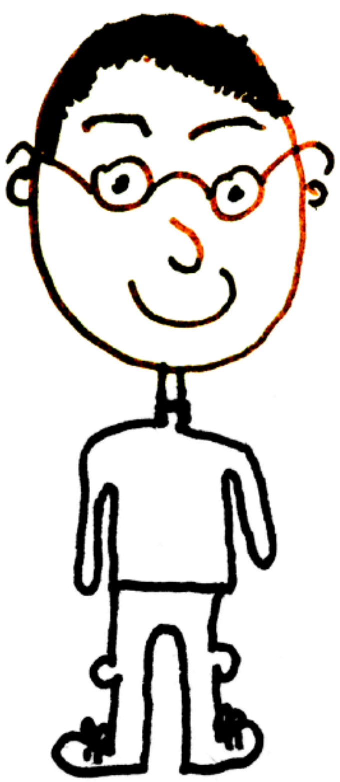
Receiver

σ_m

Signer

(sk, pk)

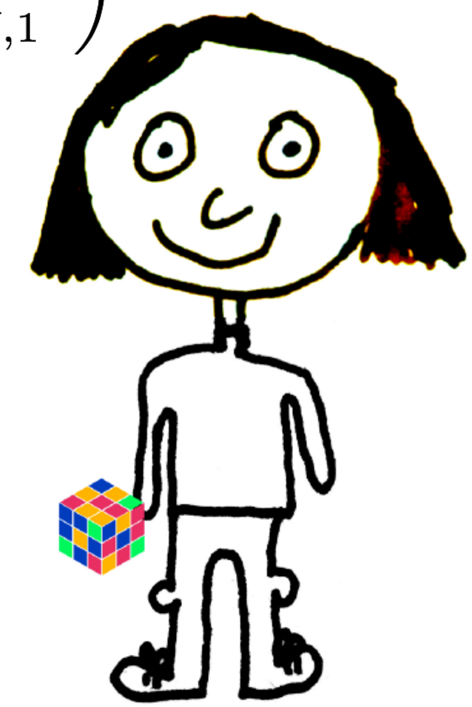
Blind Homomorphic Signatures



Receiver

m

$$\begin{pmatrix} \sigma_{1,0} & \sigma_{2,0} & \cdots & \sigma_{N,0} \\ \sigma_{1,1} & \sigma_{2,1} & \cdots & \sigma_{N,1} \end{pmatrix}$$



Signer

Blind Homomorphic Signatures

Receiver

m

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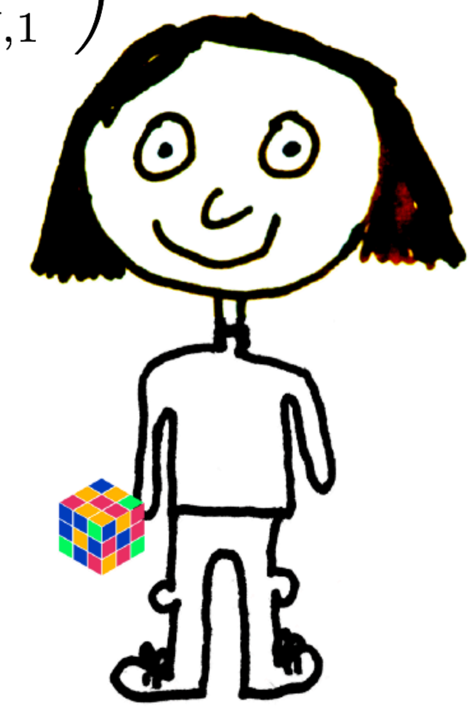
$$\xleftrightarrow{\text{2-out-1 OT}}$$

$$\vdots$$

$$\xleftrightarrow{\text{2-out-1 OT}}$$

[PVW08]



Signer

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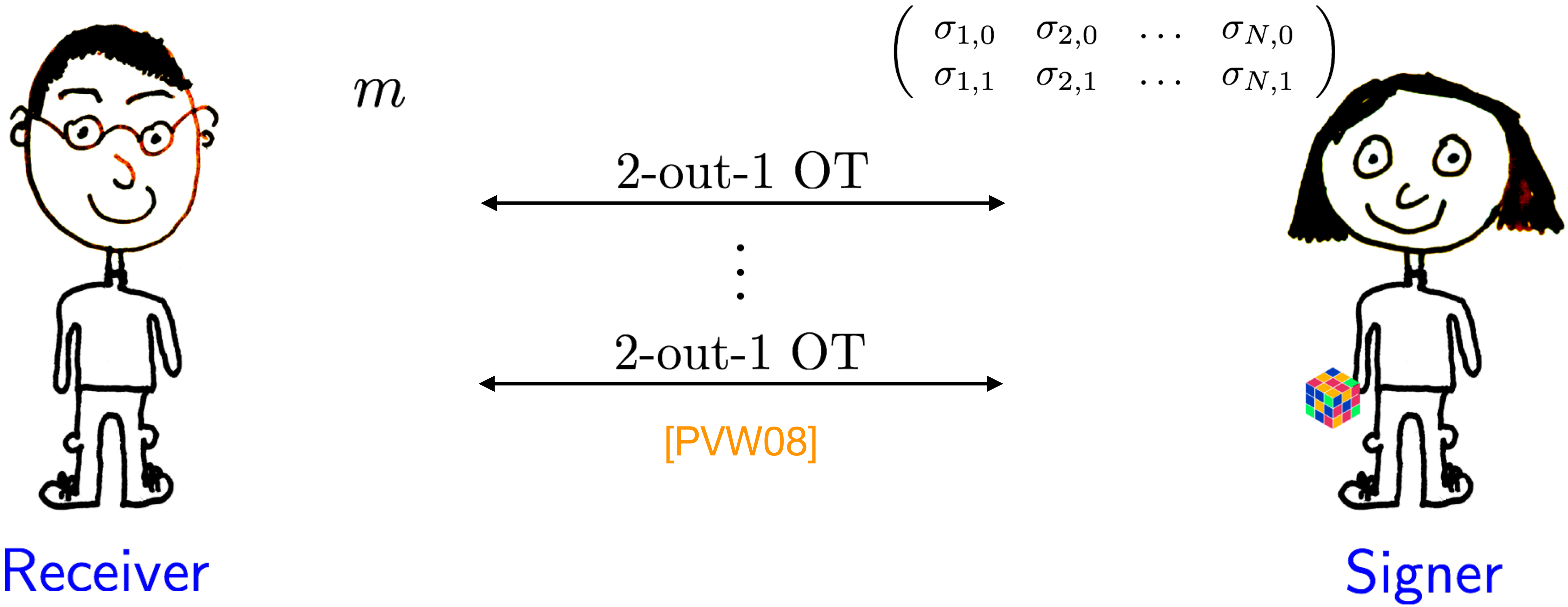
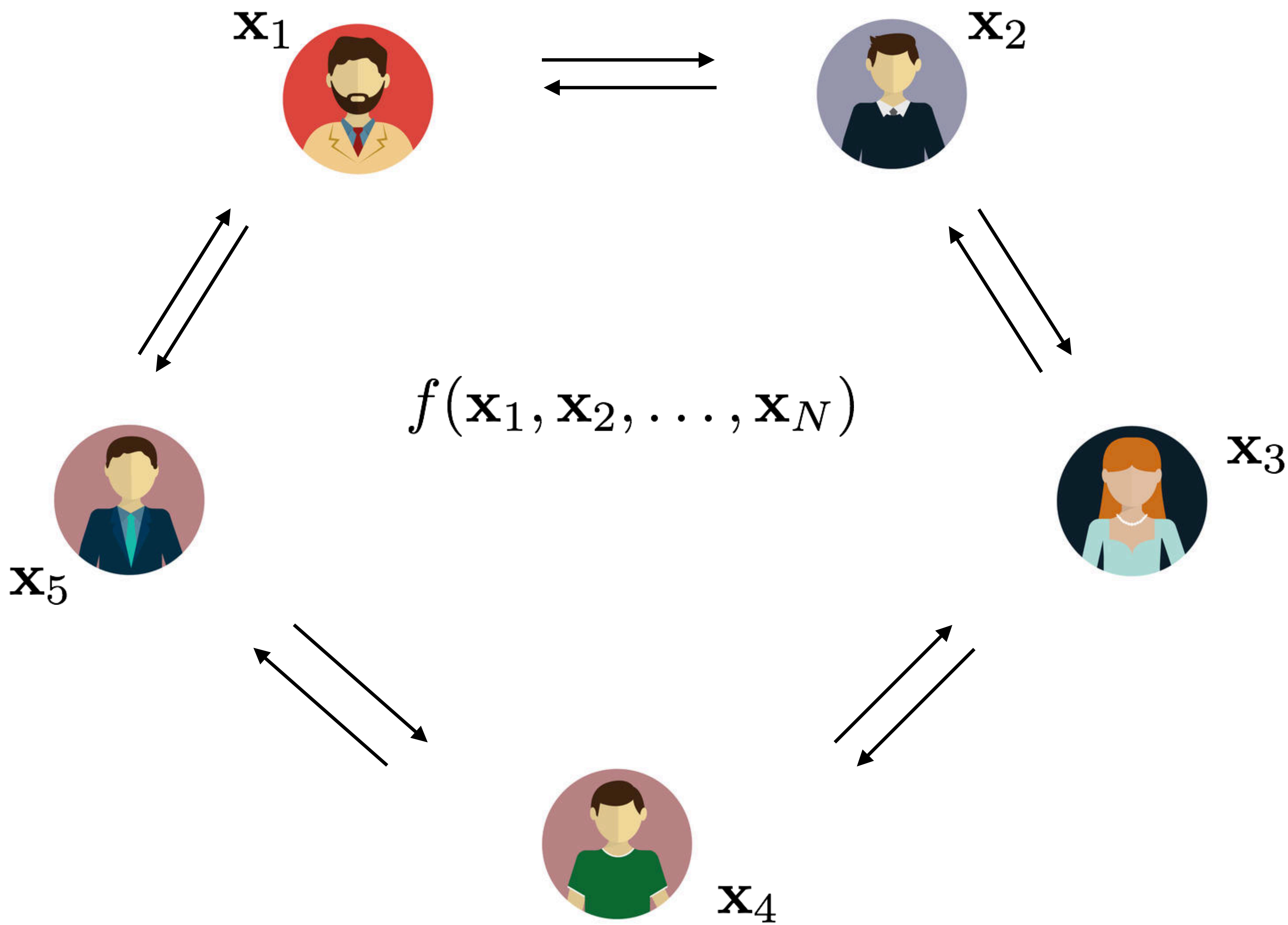


Must account for:

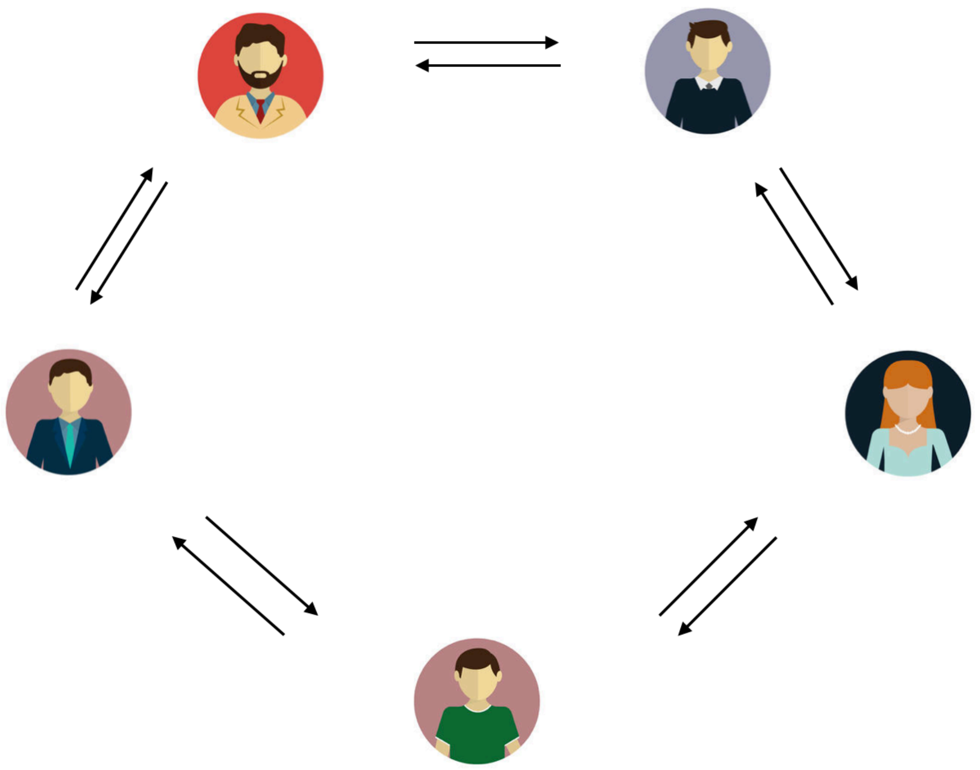
- Arbitrary abort attacks
- Maliciously generated signatures
- Guarantee context-hiding even when signer has signing key
- …

Security against malicious adversaries in MPC



Security against malicious adversaries in MPC

GMW Compiler:
Semi-Honest to Malicious security



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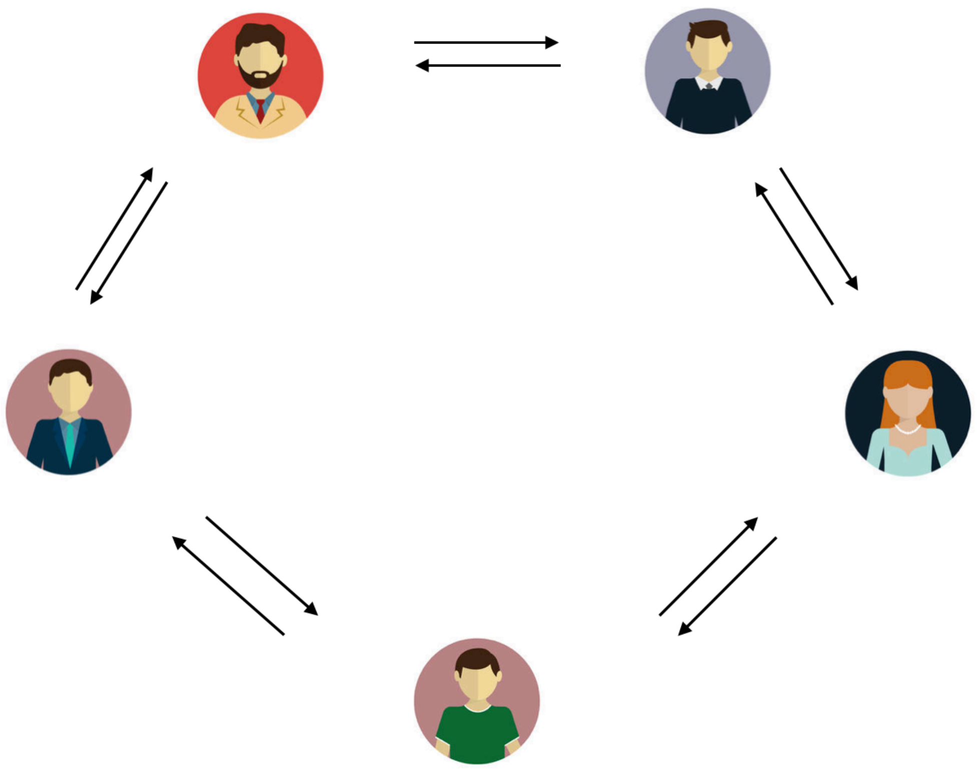
At every step, each party proves that they are following protocol.



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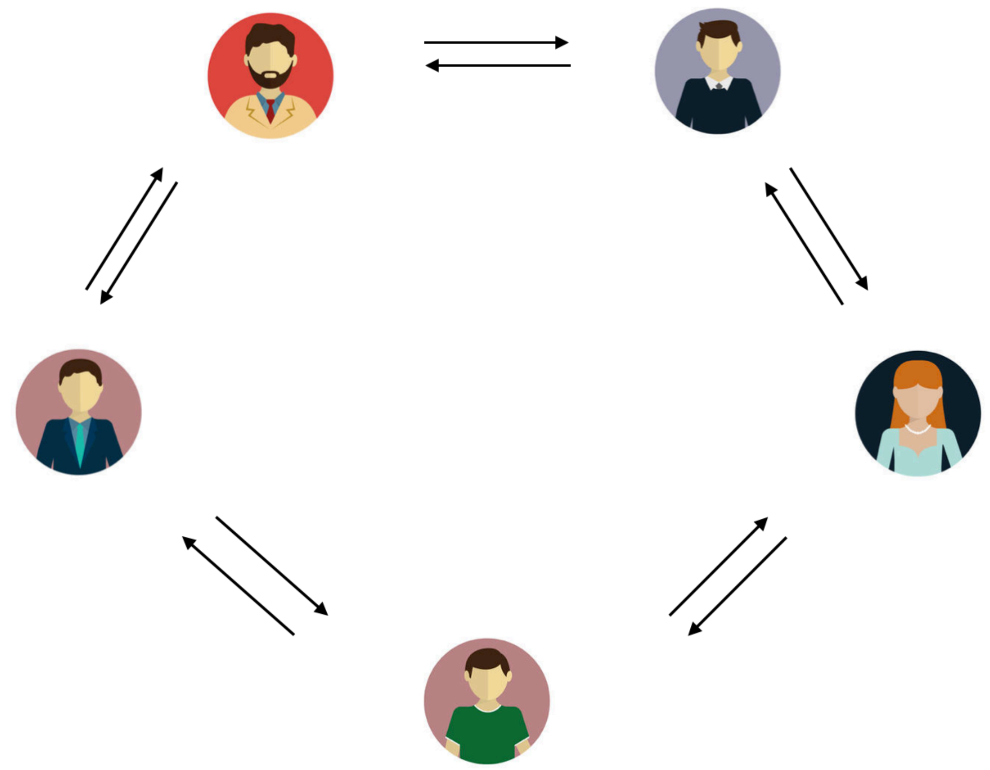
Semi-honest + NIZK = Malicious
- Preserves round complexity



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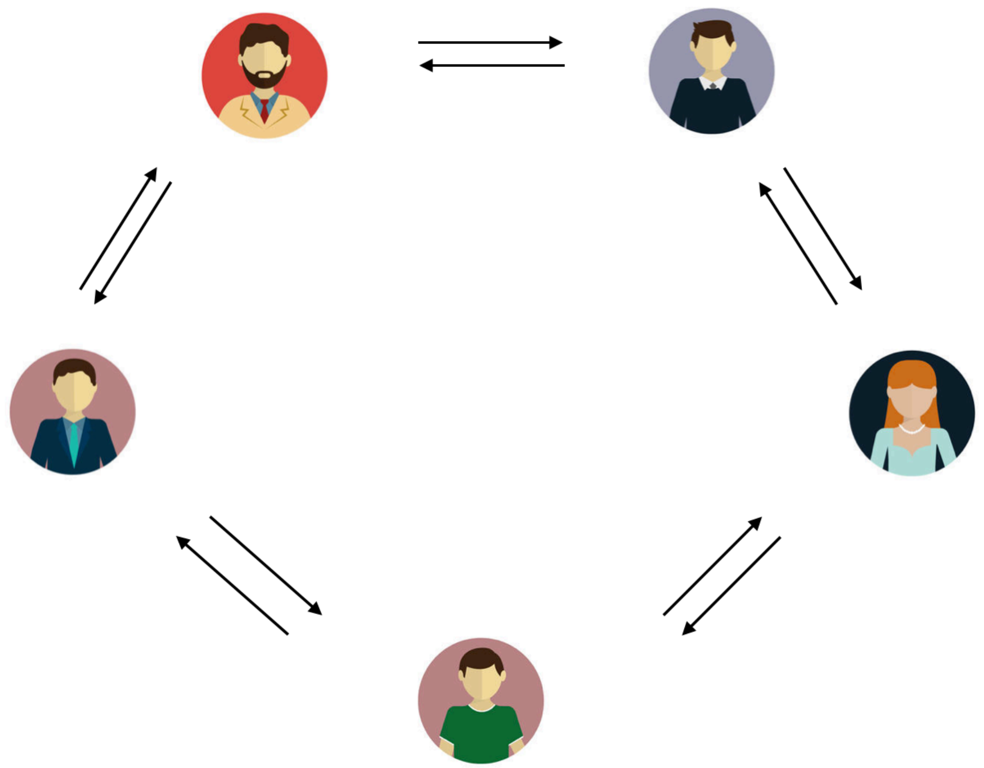
Semi-honest + NIZK w/preprocessing = Malicious
- Preprocessing step done once



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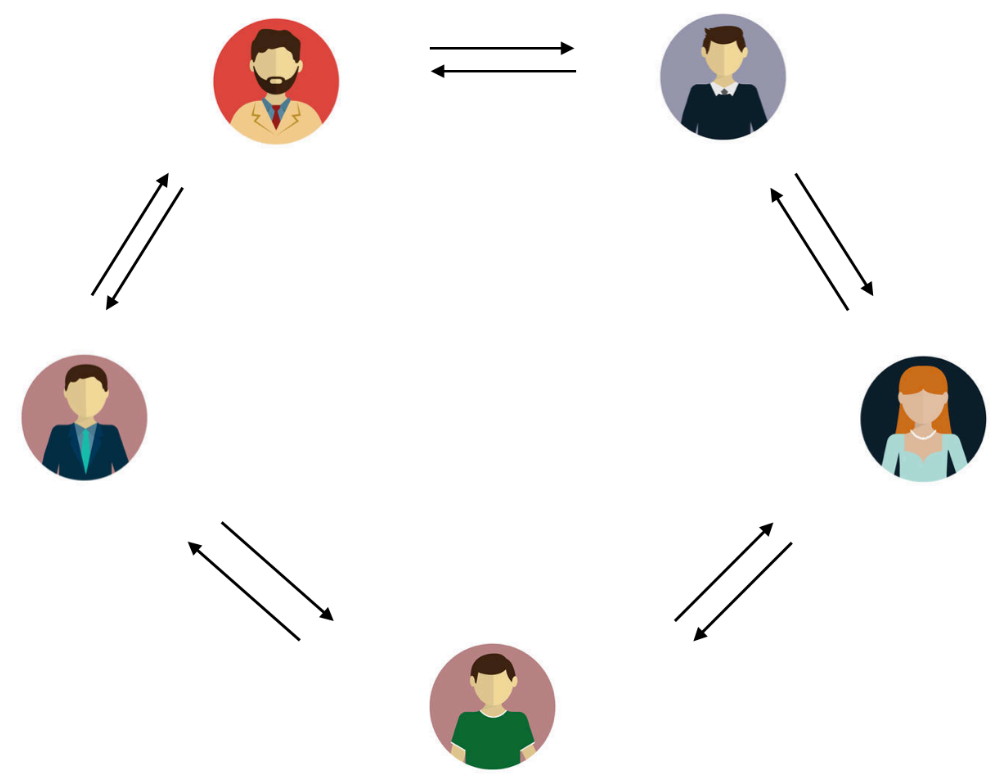

Semi-honest + NIZK w/preprocessing = Malicious
- Preprocessing step done once

Good: Rely on lattices + small communication size

To Summarize…

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1. LWE-based NIZK arguments in preprocessing (multi-theorem)

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Thanks!