Lockable Obfuscation

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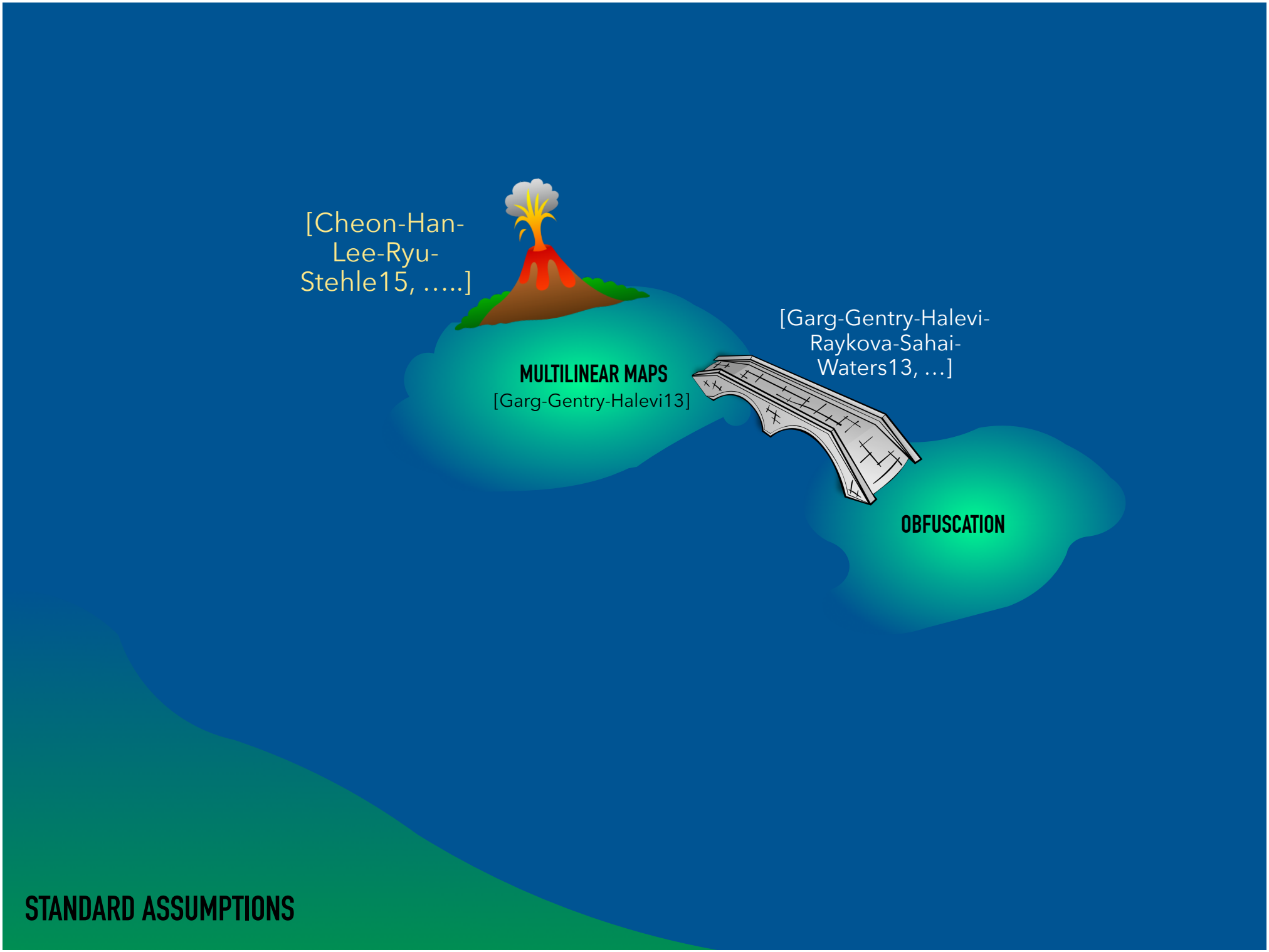
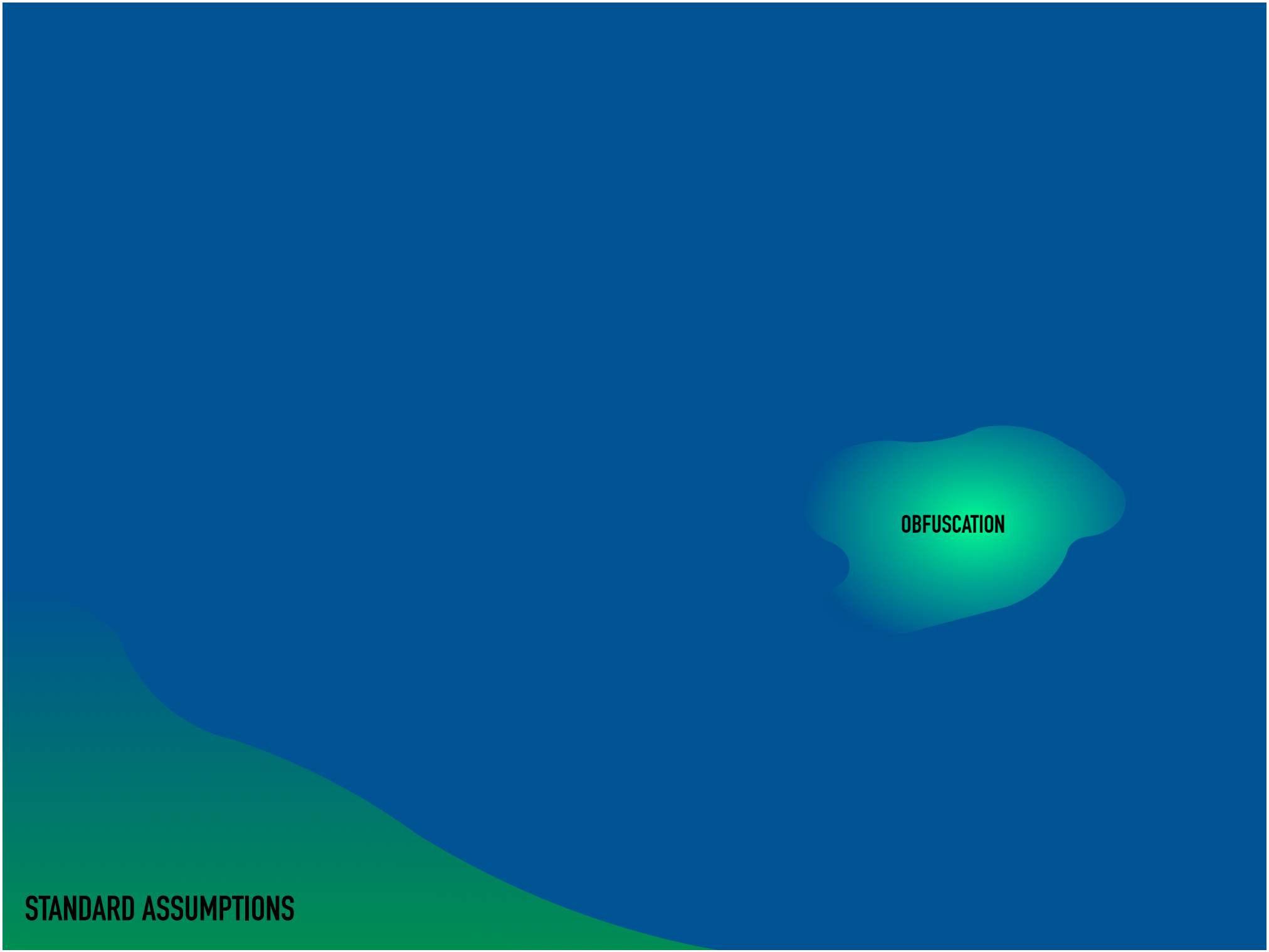
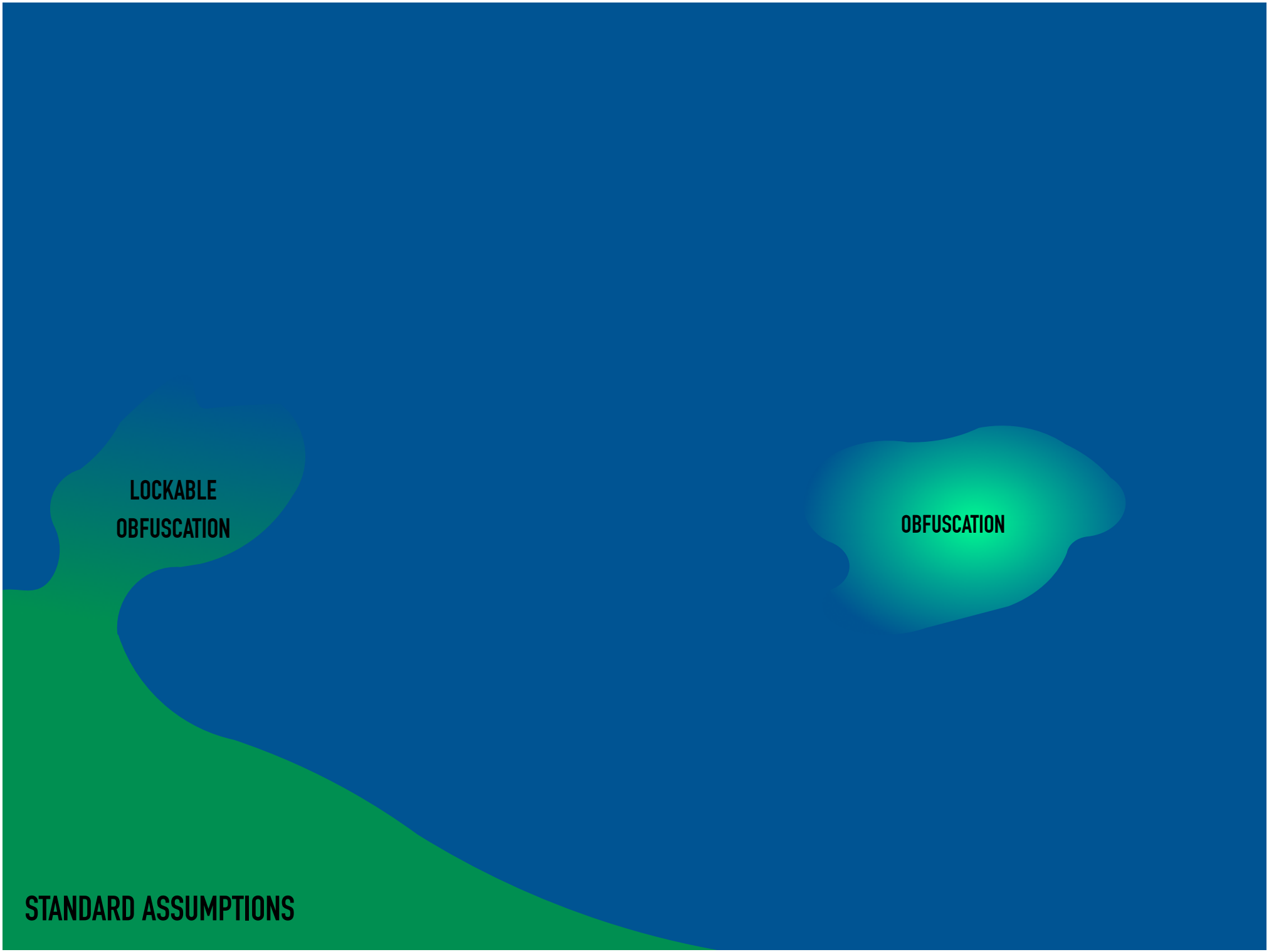
Universal Samplers
Traitor Tracing
Multiparty Key Exchange
Private Constrained PRFs
Deniable Encryption
Functional Encryption
RO uninstantiability
Trapdoor Permutations
INDISTINGUISHABILITY OBFUSCATION
[Garg-Gentry-Halevi-Raykova-Sahai-Waters13]
Garbled RAM
Circular Security separations
Witness Encryption
2-round MPC
PPAD hardness
FHE


[Cheon-Han-Lee-Ryu-Stehle15, ...]
MULTILINEAR MAPS
[Garg-Gentry-Halevi13]
[Garg-Gentry-Halevi-Raykova-Sahai-Waters13, …]
OBFUSCATION
STANDARD ASSUMPTIONS


OBFUSCATION
STANDARD ASSUMPTIONS

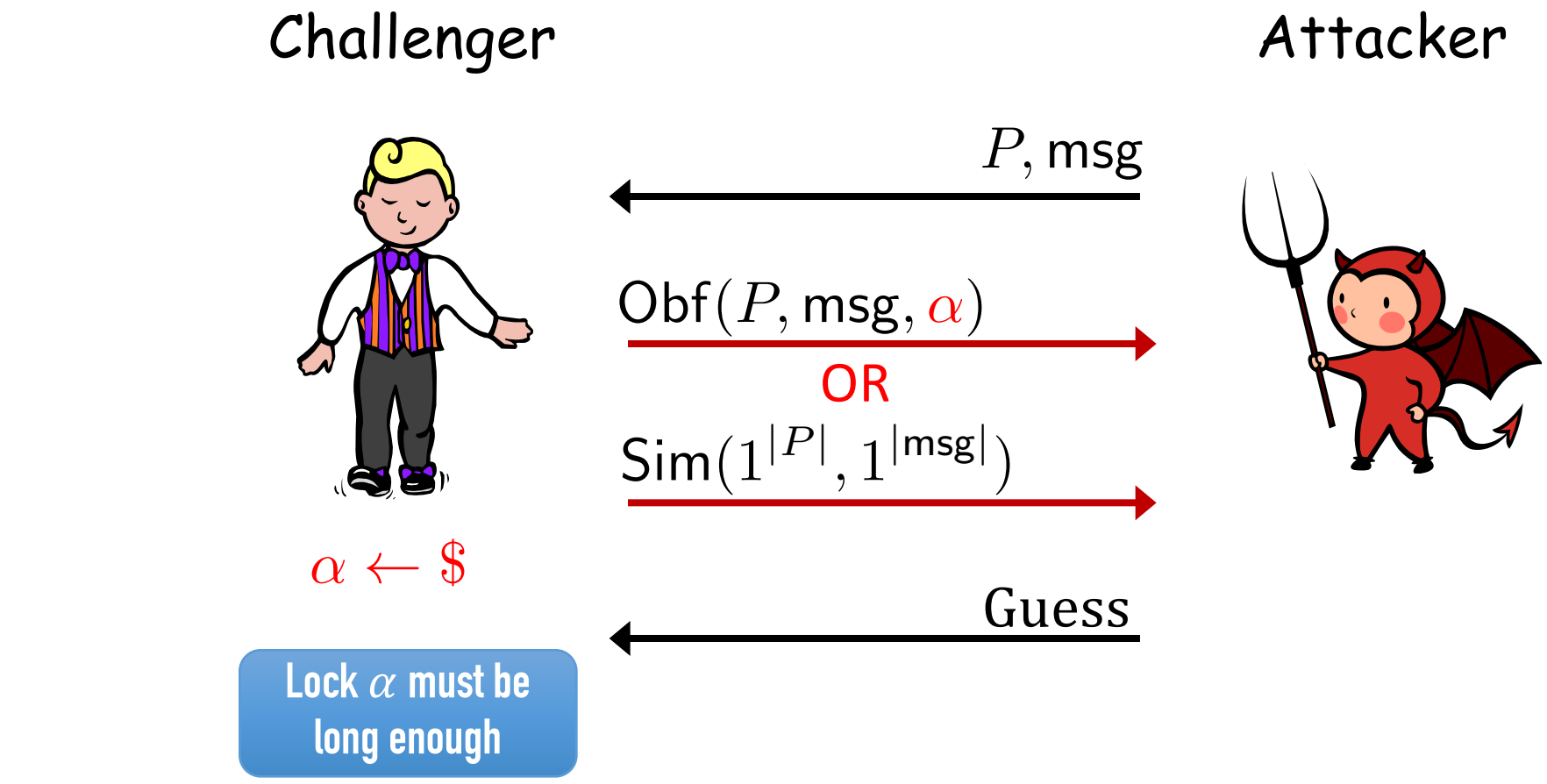

LOCKABLE
OBFUSCATION
OBFUSCATION
STANDARD ASSUMPTIONS

Lockable Obfuscation

- $\mathsf{Obf}(1^\lambda, P, \mathsf{msg}, \alpha) \rightarrow \widetilde{P}$
- $\mathsf{Eval}(\widetilde{P}, x) \rightarrow \mathsf{msg}$ or $\perp$

- Correctness:
  - If $P(x) = \alpha$, $\mathsf{Eval}(\widetilde{P}, x) = \mathsf{msg}$.
  - Else $\mathsf{Eval}(\widetilde{P}, x) = \perp$.

Lockable Obfuscation: Security



Our Result

α could be from high entropy dist

Concurrent Work
[WichsZirdelis17]

- Lockable Obfuscation
  - All poly sized circuits*
  - Secure under LWE

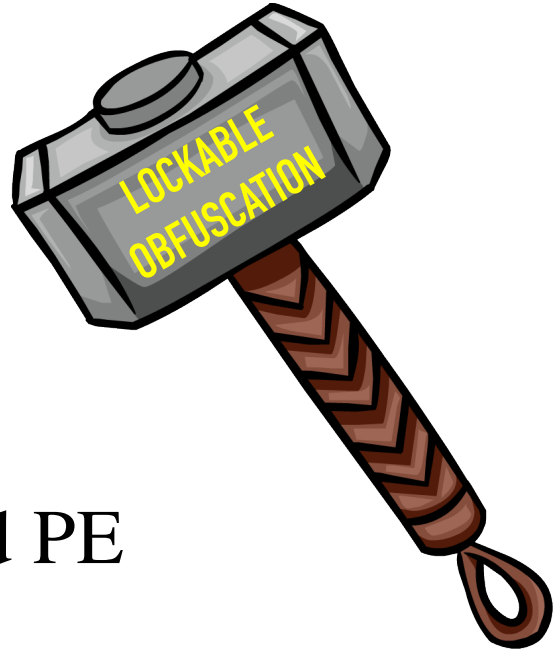
- Applications
  - Attribute-Based Encryption → 1-sided Predicate Encryption
  - Broadcast Encryption → Anonymous Broadcast Encryption
  - Witness Encryption → Reject Indistinguishability Obfuscator (riO)
  - Circular Security Separations (Bit Encryption, Unbounded, …)
  - Random Oracle Uninstantiability (Fujisaki-Okamoto, …)
  - ……

This Talk

Part I: Applications

Part II: Building Lockable Obfuscation from LWE

This Talk



Part I: Applications

- Upgrading ABE to 1-sided PE

Part II: Building Lockable Obfuscation from LWE

Attribute Based Encryption (ABE)

[SahaiWaters05]

Predicate Encryption (PE)

[KatzSahaiWaters08,BonehWaters07]

$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{mpk}, m, x)$ $\qquad$ $\mathsf{sk}_{f_1}, \ldots, \mathsf{sk}_{f_q}$

Lockable Obfuscation

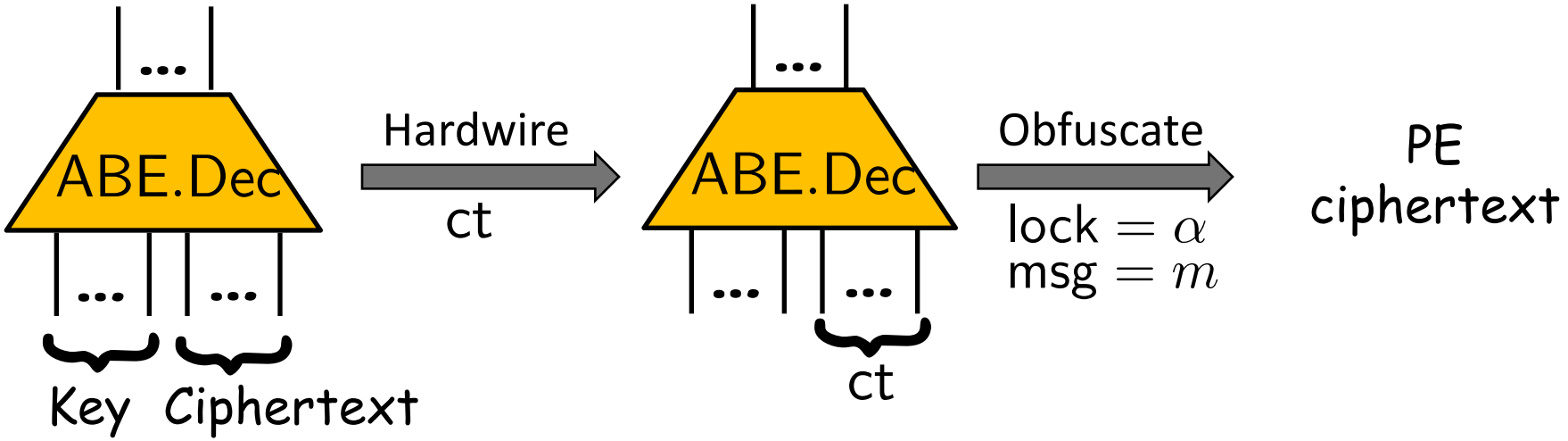
	Attribute Based Encryption	*1-sided* Predicate Encryption	Predicate Encryption
$\forall i,\ f_i(x) = 0$	m hidden x may not be	m hidden x hidden	m hidden x hidden
$\exists i,\ f_i(x) = 1$	x may not be	x may not be	x hidden

Upgrading ABE to “1-sided” PE

- Setup, KeyGen *unchanged*

Upgrading ABE to "1-sided" PE

- $Enc(\text{message } m, \text{attribute } x)$
  - Choose random lock α
  - Compute ct $\leftarrow$ ABE.Enc(α, x)

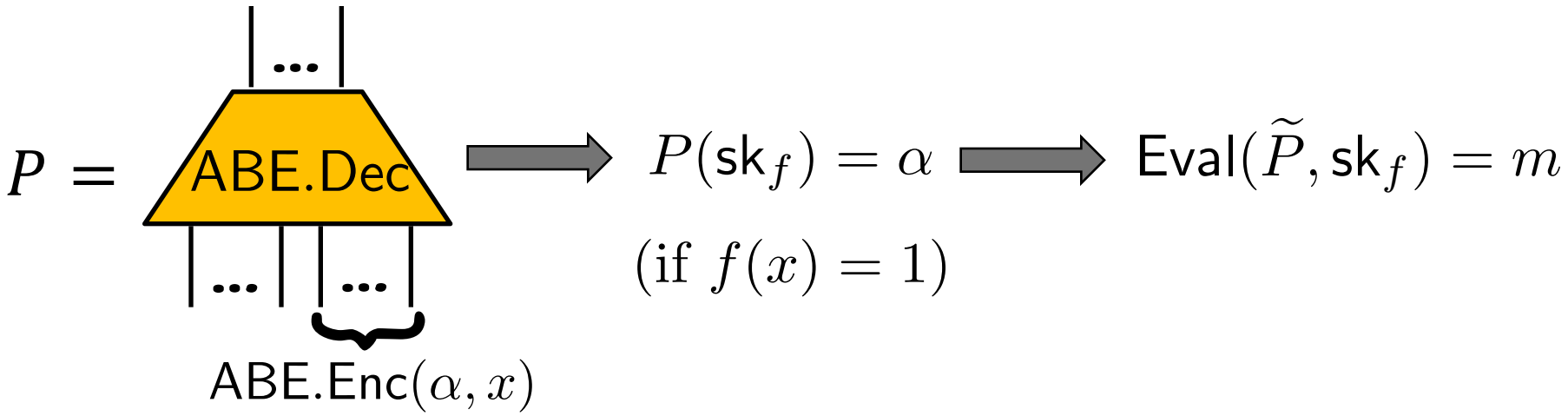


Upgrading ABE to "1-sided" PE

- *Enc*(message m, attribute x)
  - Choose random lock α
  - Compute $\mathsf{ct} \leftarrow \mathsf{ABE.Enc}(\alpha, x)$
  - Let $P = \mathsf{ABE.Dec}(\cdot, \mathsf{ct})$
  - Compute $\widetilde{P} \leftarrow \mathsf{Obf}(P, m, \alpha)$
  - Output $\widetilde{P}$

Upgrading ABE to "1-sided" PE

- $Dec(\text{key } sk_f, \text{ciphertext } \tilde{P})$
  - Output $\mathsf{Eval}(\widetilde{P}, \mathsf{sk}_f)$



Proof Overview

- Choose random lock α
- Compute $\mathsf{ct} \leftarrow \mathsf{ABE.Enc}(\alpha, x)$
- Let $P = \mathsf{ABE.Dec}(\cdot, \mathsf{ct})$
- Compute $\widetilde{P} \leftarrow \mathsf{Obf}(P, m, \alpha)$
- Output $\widetilde{P}$

$\approx$ (Using ABE security)

$\vdots$

- Compute $\mathsf{ct} \leftarrow \mathsf{ABE.Enc}(0, x)$

$\vdots$

Proof Overview

- Choose random lock α
- Compute $\mathsf{ct} \leftarrow \mathsf{ABE.Enc}(0, x)$
- Let $P = \mathsf{ABE.Dec}(\cdot, \mathsf{ct})$
- Compute $\widetilde{P} \leftarrow \mathsf{Obf}(P, m, \alpha)$
- Output $\widetilde{P}$

$\approx$ (Using Lockable Obfuscator)

- Output $\widetilde{P} \leftarrow \mathsf{Sim}(\cdots)$

1-Cycle Tester

- Setup : Choose key pair $(\mathsf{pk}, \mathsf{sk})$, string s
  Compute aux = obfuscation of

iO

Constants: Key sk, String s
Input: Ciphertext ct

– Compute $(m_1, m_2) = \mathsf{Dec}(\mathsf{sk}, \mathsf{ct})$.
– If $\mathrm{PRG}(m_2) = \mathrm{PRG}(s)$, output 1.

Lockable

$\alpha = s, \mathsf{msg} = 1$

Constants: Key sk
Input: Ciphertext ct

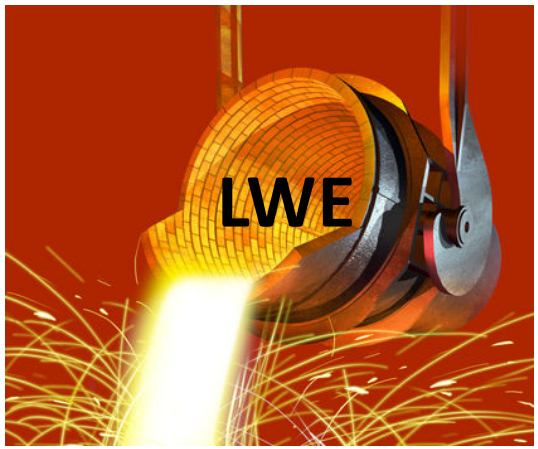
– Compute $(m_1, m_2) = \mathsf{Dec}(\mathsf{sk}, \mathsf{ct})$.
– Output m_2.

Output $\mathsf{pk}' = (\mathsf{pk}, \mathrm{aux}), \mathsf{sk}' = (\mathsf{sk}, s)$.

This Talk

Part I: Applications

Part II: Building Lockable Obfuscation from LWE



Lattice Trapdoors [GentryPeikertVaikuntanathan08,…]

$\texttt{TrapGen}(1^n, 1^m, q) \rightarrow (\mathbf{A}, T_{\mathbf{A}})$ $\quad\quad$ $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$

$\texttt{SamplePre}(\mathbf{A}, T_{\mathbf{A}}, \sigma, \mathbf{Z}) \rightarrow \mathbf{U}$ $\quad\quad$ $\mathbf{A} \cdot \mathbf{U} = \mathbf{Z}$ and $||\mathbf{U}||$ is small

Construction: Breaking It Down

- Step I: Log-depth circuits
  1-bit messages
- Step II: General circuits

Why log-depth?

Barrington's Theorem

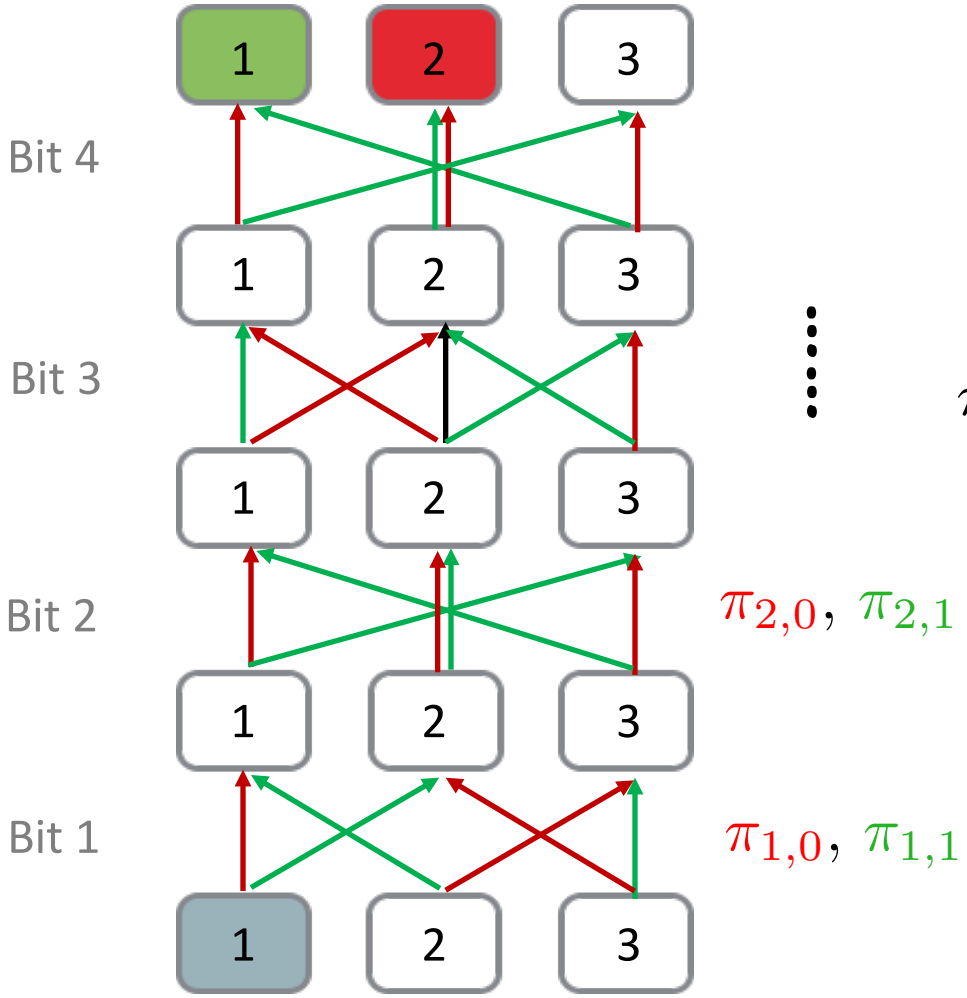
Log-depth circuits can be expressed as poly-length *permutation* branching programs of width 5.

Techniques from works on circular security

CIRCULAR SECURITY SEPARATIONS FOR SYMMETRIC-KEY BIT ENCRYPTION FROM LWE

G, KOPPULA, WATERS

Leveled Permutation Branching Programs



Read-once BP on 4-bit inputs

Say Width-3, Length-5

$\pi_{j,b} : [3] \to [3]$ specifies transitions

Simplifications

Read-once Permutation BPs

ℓ-bit input and ℓ-bit output

Length $\ell + 1$, Width 3

Fixed input-bit selector

"small" entries/ error terms → 0

Preview

Obfuscated Program: $\widetilde{P} = \begin{cases} \mathbf{M}_{1,1}^{(\ell)}, & (\mathbf{C}_{1,0}^{(\ell)}, \mathbf{C}_{1,1}^{(\ell)}), \ldots, (\mathbf{C}_{\ell,0}^{(\ell)}, \mathbf{C}_{\ell,1}^{(\ell)}) \\ \vdots & \\ \mathbf{M}_{1,1}^{(1)}, & (\mathbf{C}_{1,0}^{(1)}, \mathbf{C}_{1,1}^{(1)}), \ldots, (\mathbf{C}_{\ell,0}^{(1)}, \mathbf{C}_{\ell,1}^{(1)}) \end{cases}$

Encode BP computing i^{th} output bit

Starting state

State Transitions

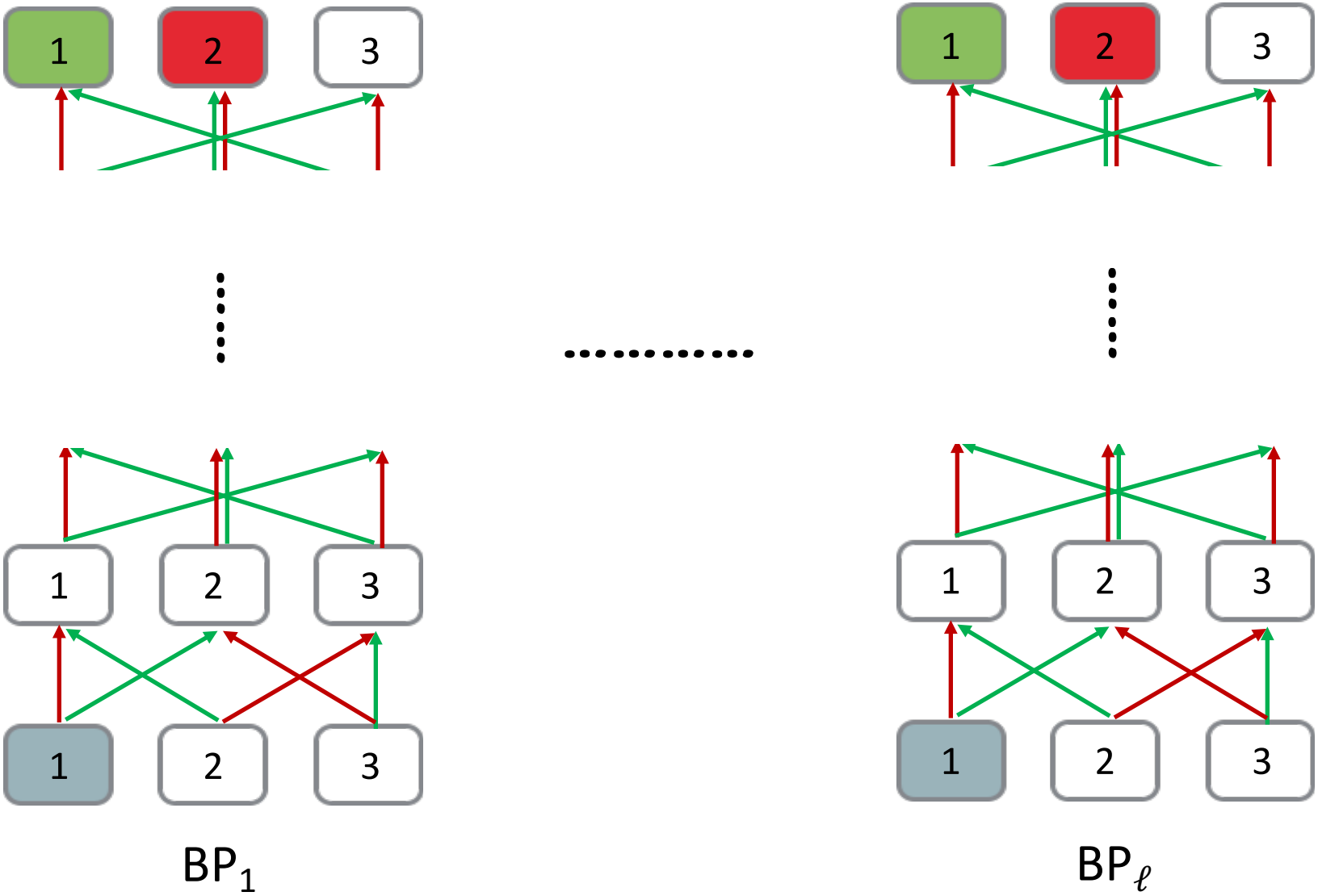
$$\mathbf{M}_{1,1}^{(i)}, \quad (\mathbf{C}_{1,0}^{(i)}, \mathbf{C}_{1,1}^{(i)}), \ldots, (\mathbf{C}_{\ell,0}^{(i)}, \mathbf{C}_{\ell,1}^{(i)})$$

$$\boxed{\mathbf{M}_{1,1}^{(i)}} \times \boxed{\mathbf{C}_{1,x_1}^{(i)}} \times \boxed{\mathbf{C}_{2,x_2}^{(i)}} \cdots\cdots\cdots \boxed{\mathbf{C}_{\ell,x_\ell}^{(i)}}$$

Sum & Test

Obf(P,α,msg)

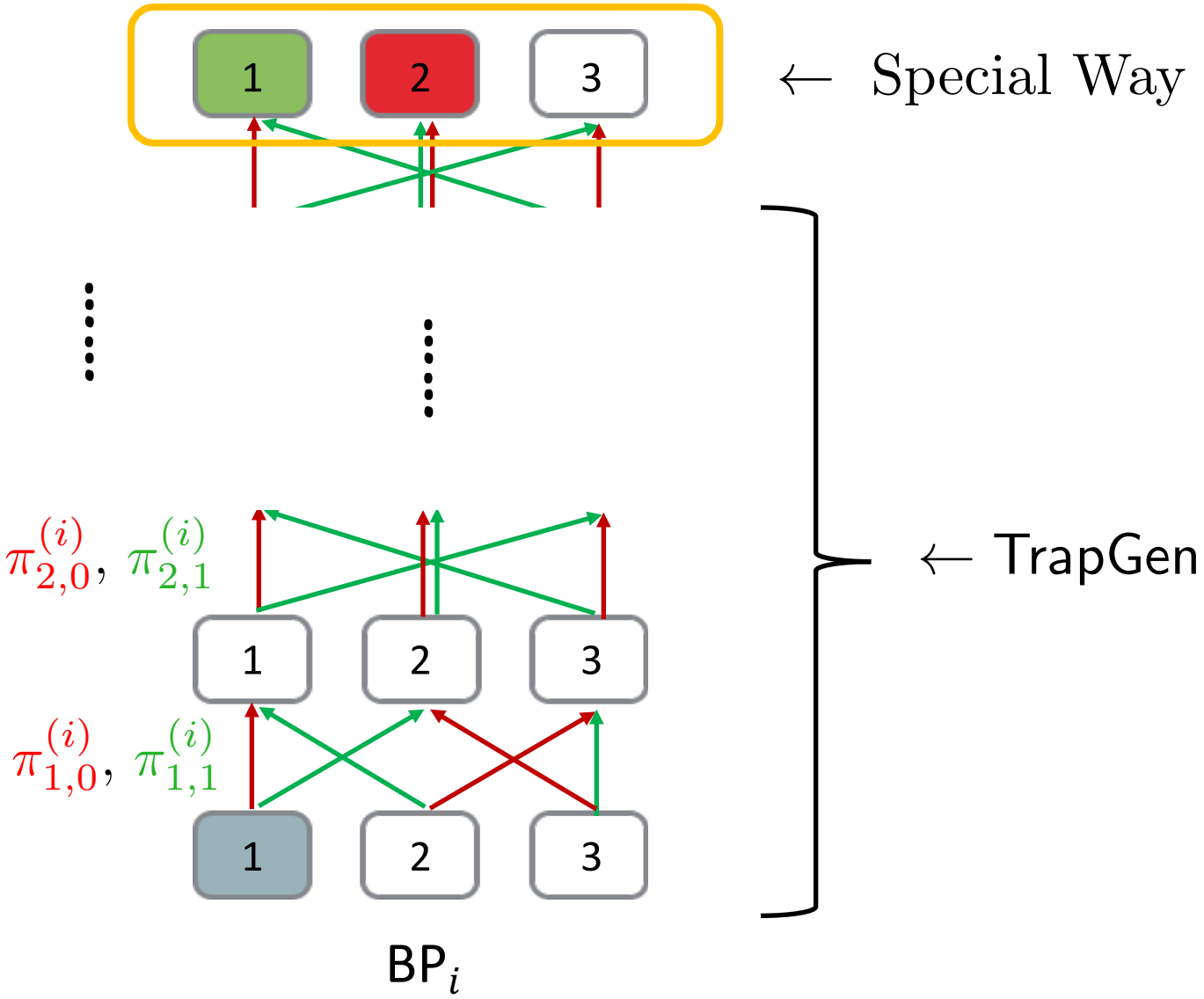
Let $P(x) = \mathsf{BP}_1(x)||\cdots||\mathsf{BP}_\ell(x)$



Obf(P,α,msg)

$$\text{Let } P(x) = \mathsf{BP}_1(x)||\cdots||\mathsf{BP}_\ell(x)$$

$$\mathsf{BP}_i(x) = (\pi^{(i)}_{1,0}, \pi^{(i)}_{1,1}), \ldots, (\pi^{(i)}_{\ell,0}, \pi^{(i)}_{\ell,1})$$



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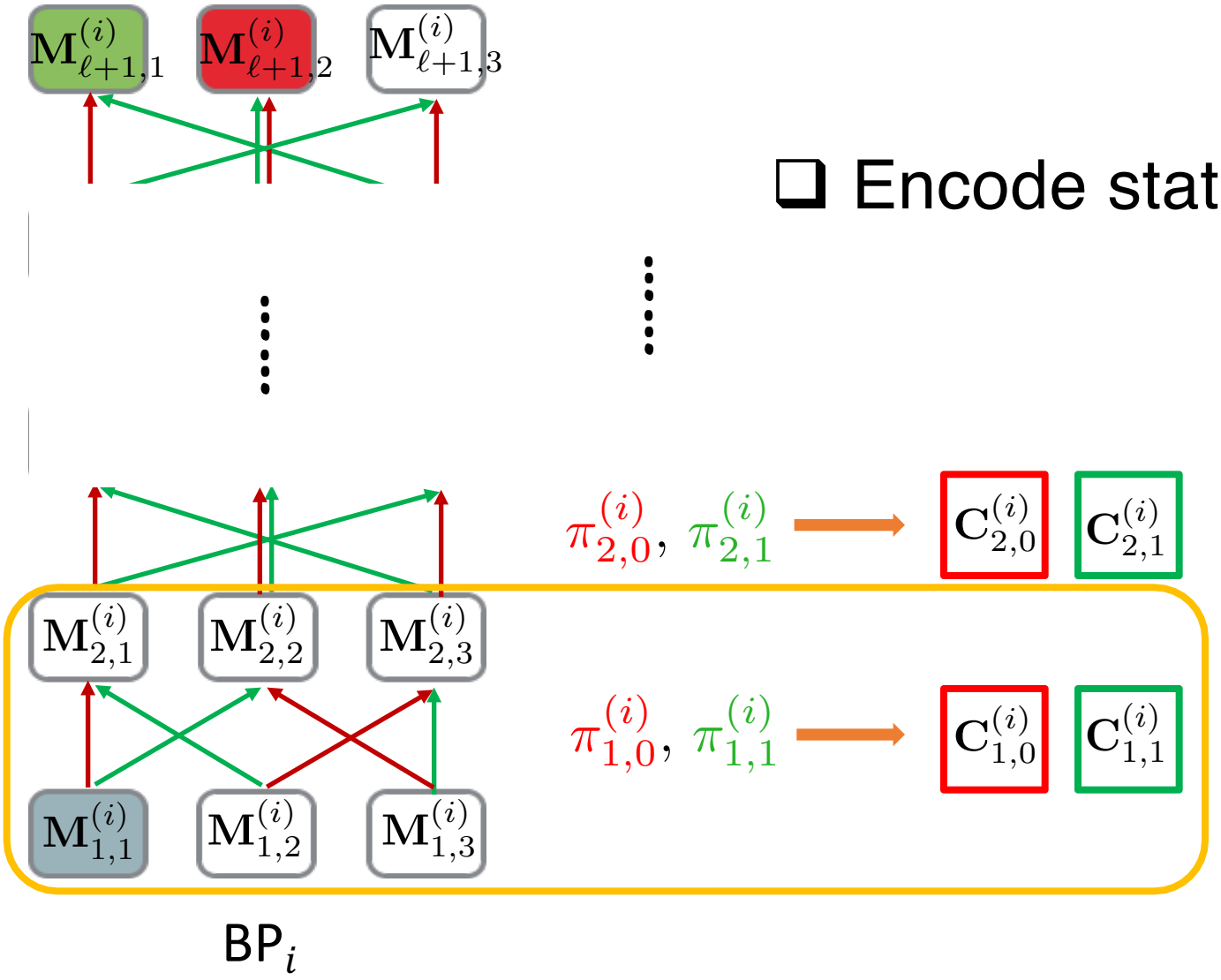
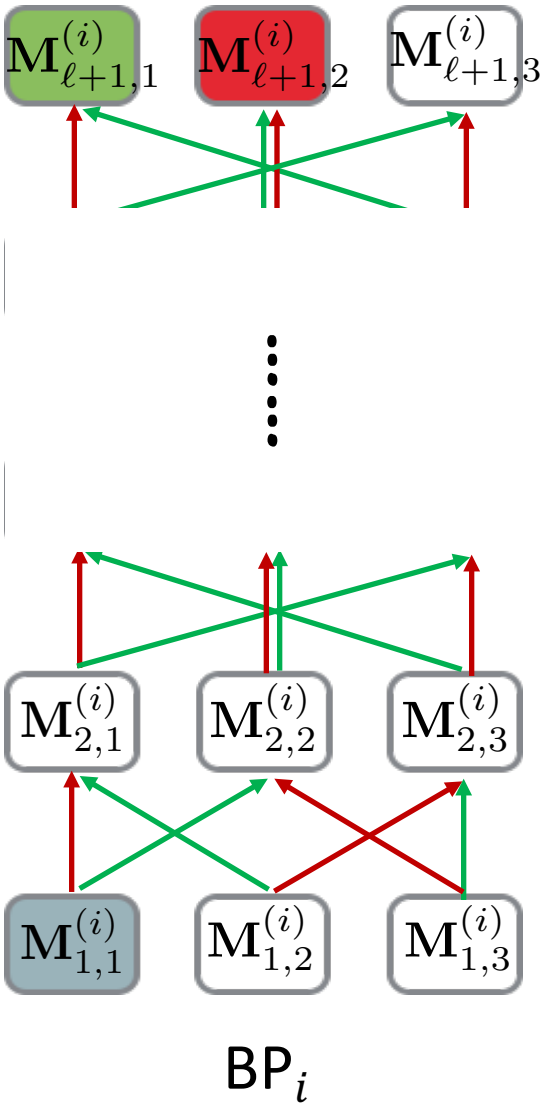


BP_i

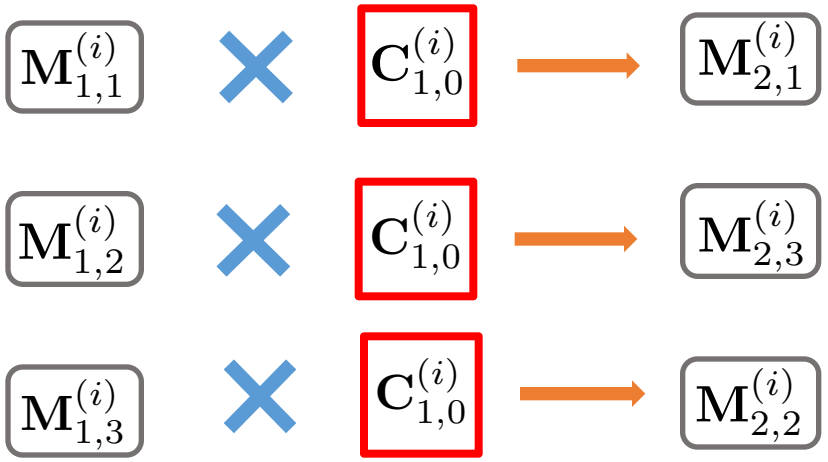
- Encode state transitions

Obf(P,α,msg)

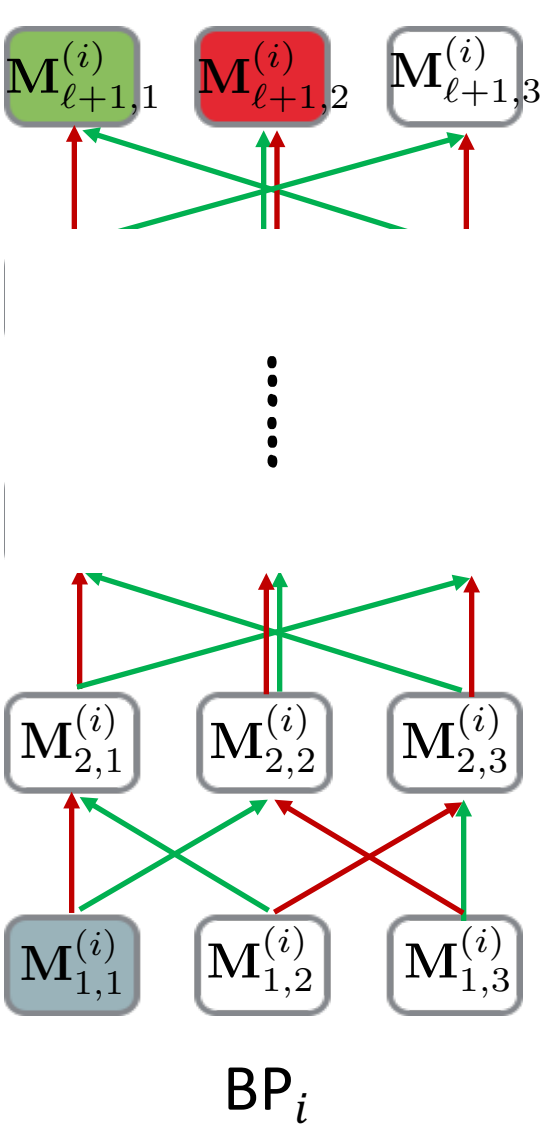
$$\mathsf{BP}_i(x) = (\pi_{1,0}^{(i)}, \pi_{1,1}^{(i)}), \ldots, (\pi_{\ell,0}^{(i)}, \pi_{\ell,1}^{(i)})$$



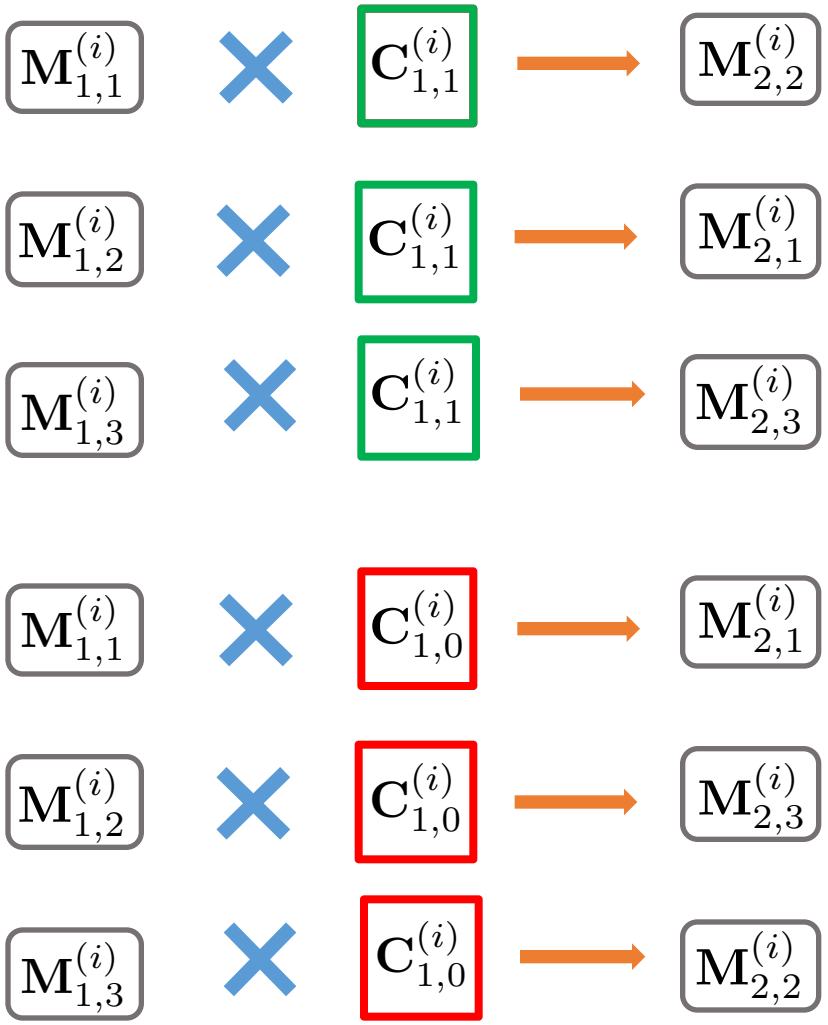
BP_i



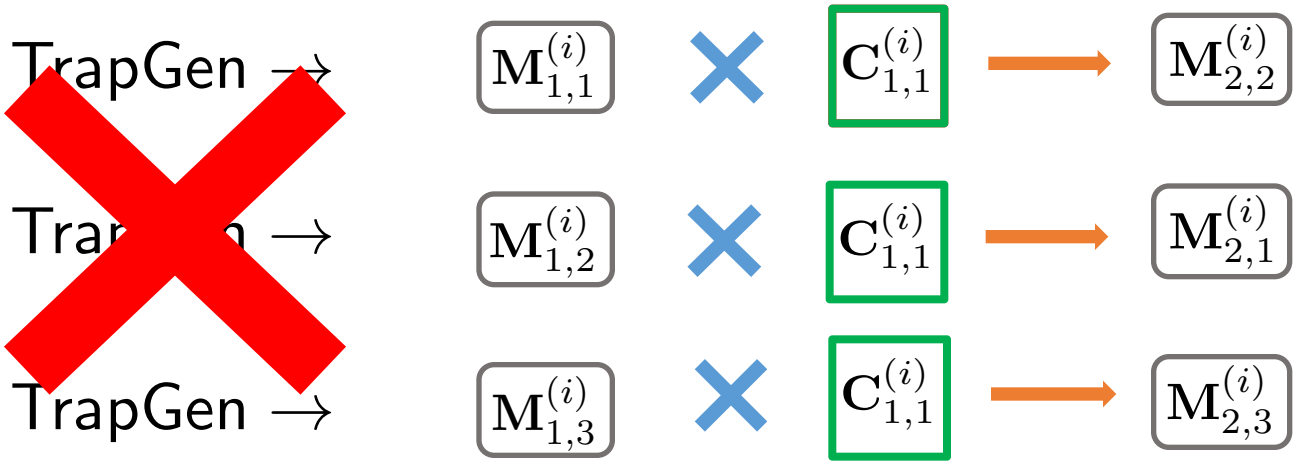
Obf(P,α,msg)



BP_i



How to SamplePre?



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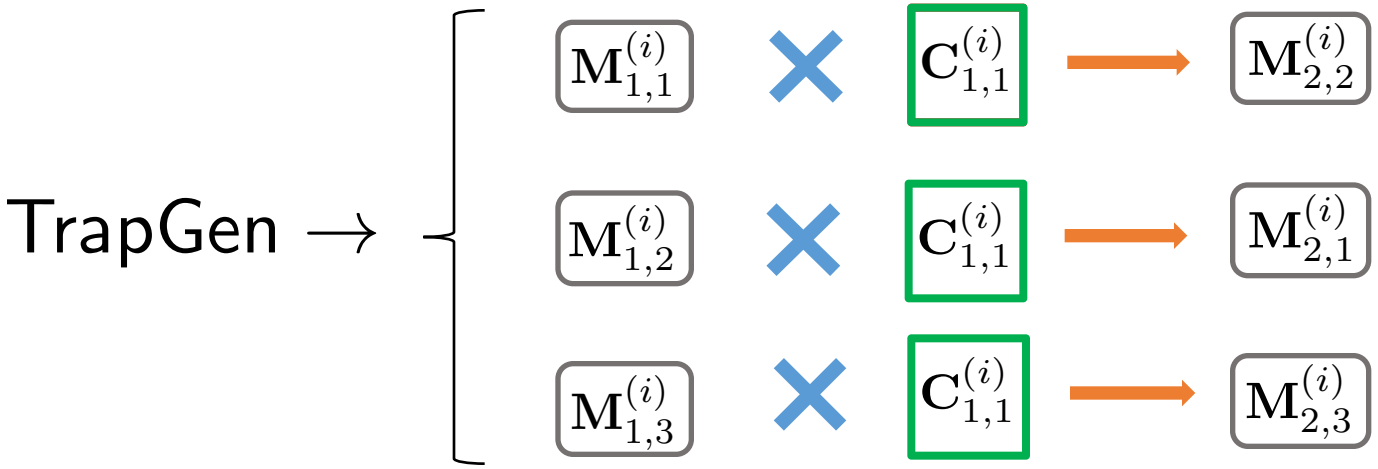


✓ Joint trapdoor generation at *each level* for every BP_i

NOTE: Cannot jointly sample for all BP_i together (Parameter issue)

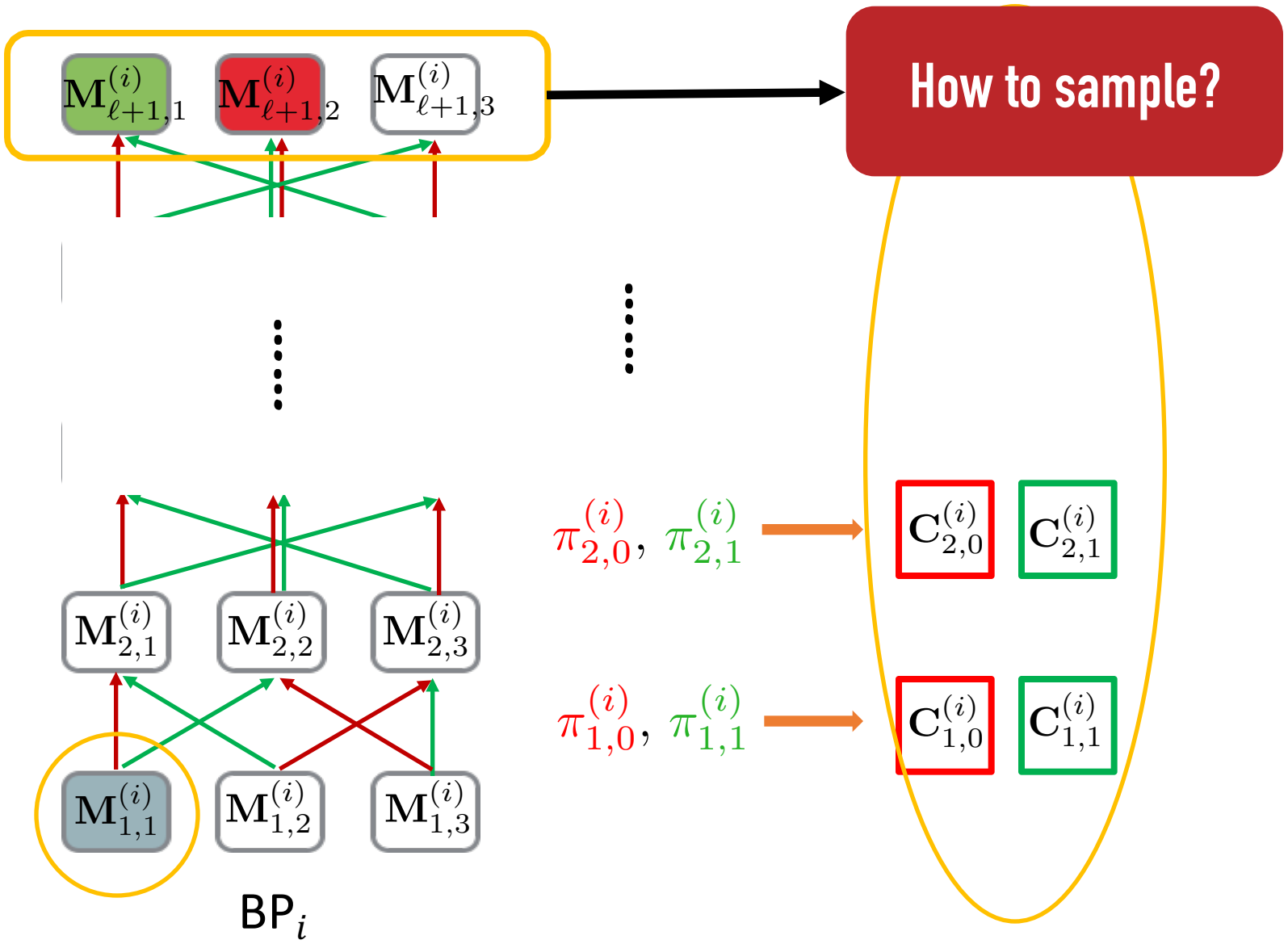
For Security

- Choose short secrets $\mathbf{S}_{j,0}, \mathbf{S}_{j,1}$ Per level j BUT shared across BP_i

$$\begin{bmatrix} \mathbf{M}^{(i)}_{1,1} \\ \mathbf{M}^{(i)}_{1,2} \\ \mathbf{M}^{(i)}_{1,3} \end{bmatrix} \xrightarrow{\mathbf{C}^{(i)}_{1,0}} \begin{bmatrix} \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,0}(1)} \\ \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,0}(2)} \\ \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,0}(3)} \end{bmatrix} \quad \begin{bmatrix} \mathbf{S}_{1,0} \cdot \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,0}(1)} \\ \mathbf{S}_{1,0} \cdot \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,0}(2)} \\ \mathbf{S}_{1,0} \cdot \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,0}(3)} \end{bmatrix} + \text{noise}$$

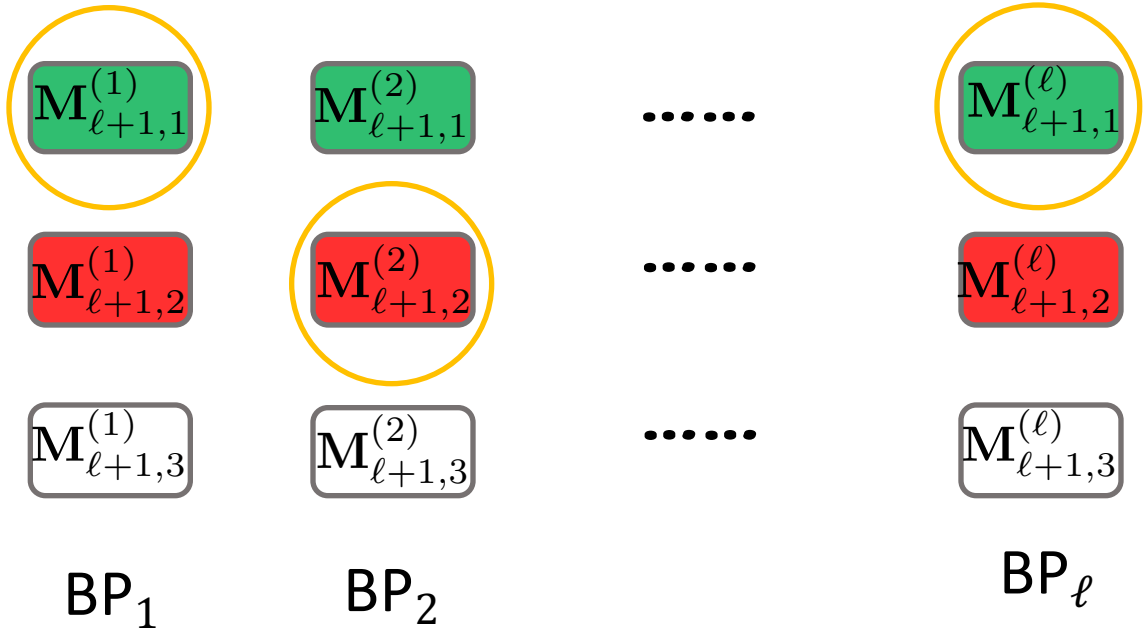
$$\begin{bmatrix} \mathbf{M}^{(i)}_{1,1} \\ \mathbf{M}^{(i)}_{1,2} \\ \mathbf{M}^{(i)}_{1,3} \end{bmatrix} \xrightarrow{\mathbf{C}^{(i)}_{1,1}} \begin{bmatrix} \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,1}(1)} \\ \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,1}(2)} \\ \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,1}(3)} \end{bmatrix} \quad \begin{bmatrix} \mathbf{S}_{1,1} \cdot \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,1}(1)} \\ \mathbf{S}_{1,1} \cdot \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,1}(2)} \\ \mathbf{S}_{1,1} \cdot \mathbf{M}^{(i)}_{2,\pi^{(i)}_{1,1}(3)} \end{bmatrix} + \text{noise}$$

Obf(P,α,msg)



Encoding Lock α and msg

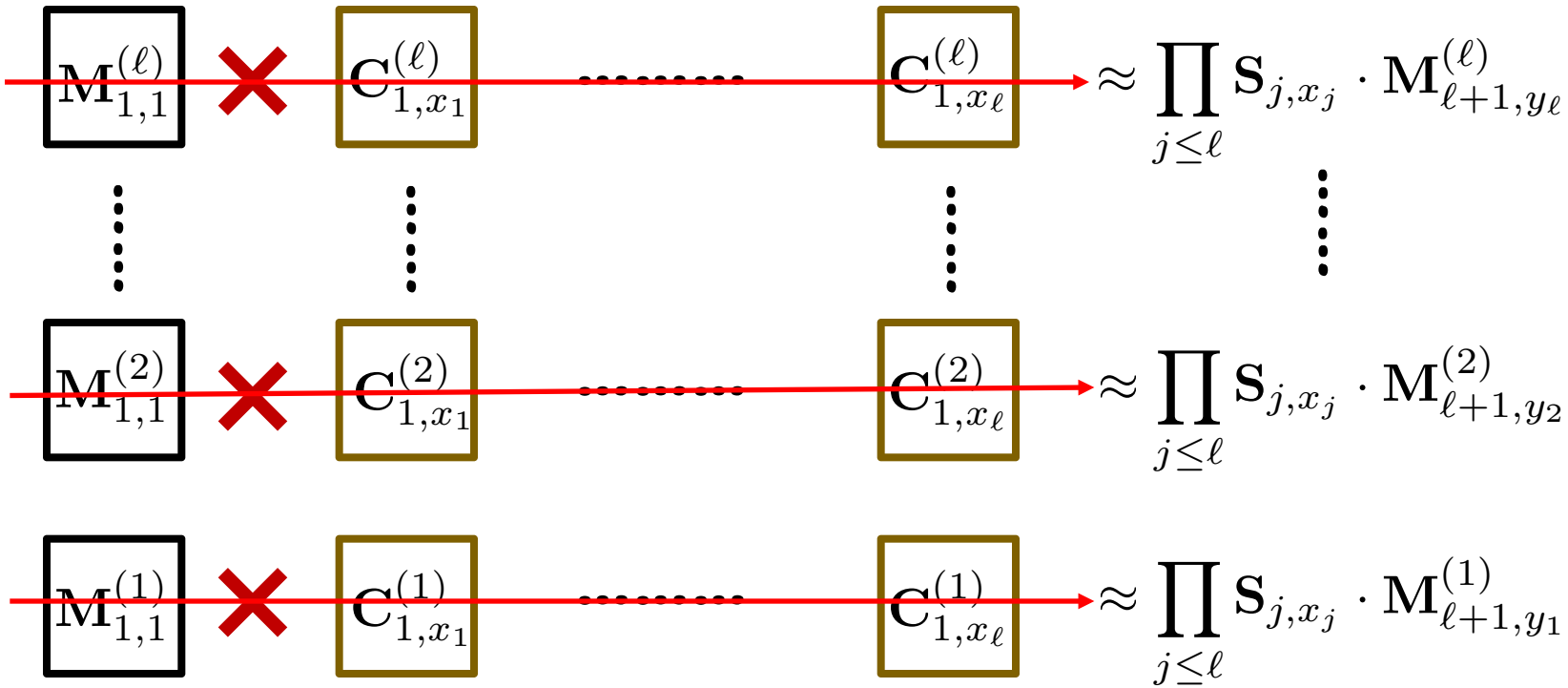
Say $\alpha = 10 \cdots 1$



$$\sum_{i \le \ell} \mathbf{M}^{(i)}_{\ell+1,\alpha_i} = \begin{cases} \mathbf{0} & \text{if } \mathsf{msg} = 0 \\ \sqrt{q} \cdot \mathbf{I} & \text{otherwise} \end{cases}$$

$\mathsf{Eval}(\widetilde{\mathrm{P}}, x)$

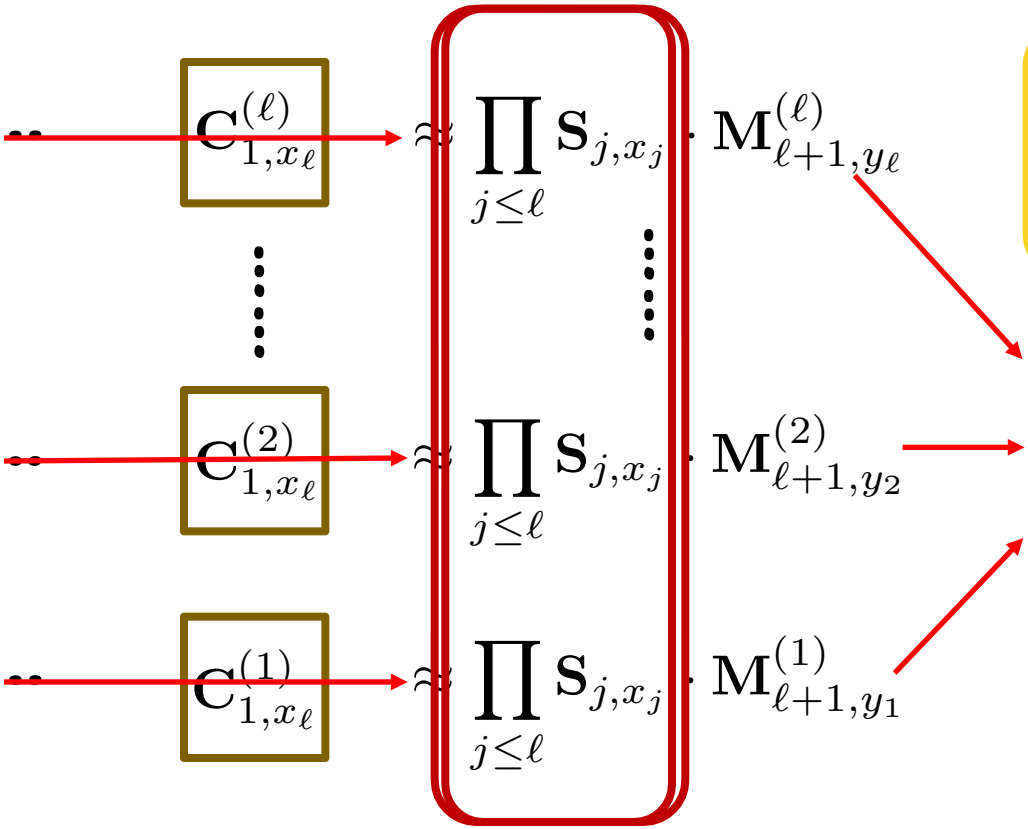
Let $y = P(x)$



If all entries 'small', output 0;
all entries 'medium', output 1;
otherwise output $\perp$

$\mathsf{Eval}(\widetilde{\mathrm{P}}, x)$

Let $y = P(x)$



If all entries 'small', output 0;
all entries 'medium', output 1;
otherwise output $\perp$

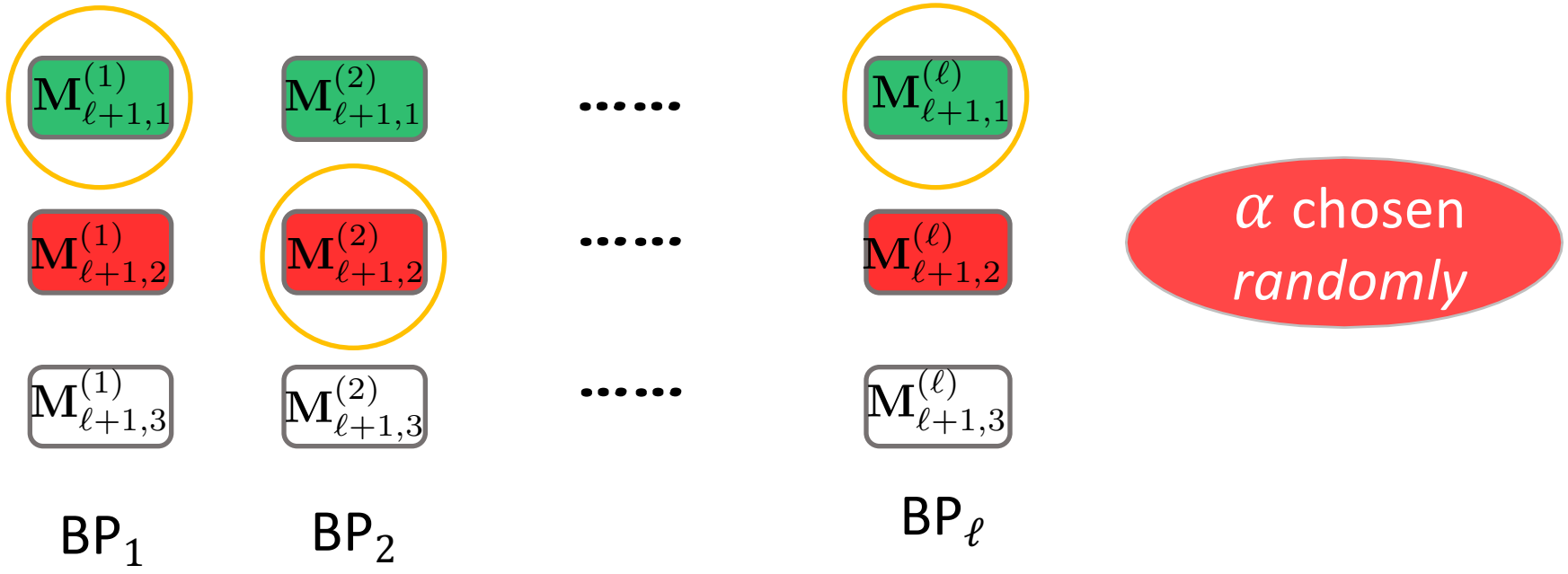
$$+ \approx \prod_{j\le\ell} \mathbf{S}_{j,x_j} \cdot \sum_{i\le\ell} \mathbf{M}^{(i)}_{\ell+1,y_i}$$

If $y = \alpha$

$$= \begin{cases} \mathbf{0} & \text{if msg} = 0 \\ \sqrt{q} \cdot \mathbf{I} & \text{otherwise} \end{cases}$$

BP_i share input-selector
S matrices reused at each level

Proof Sketch



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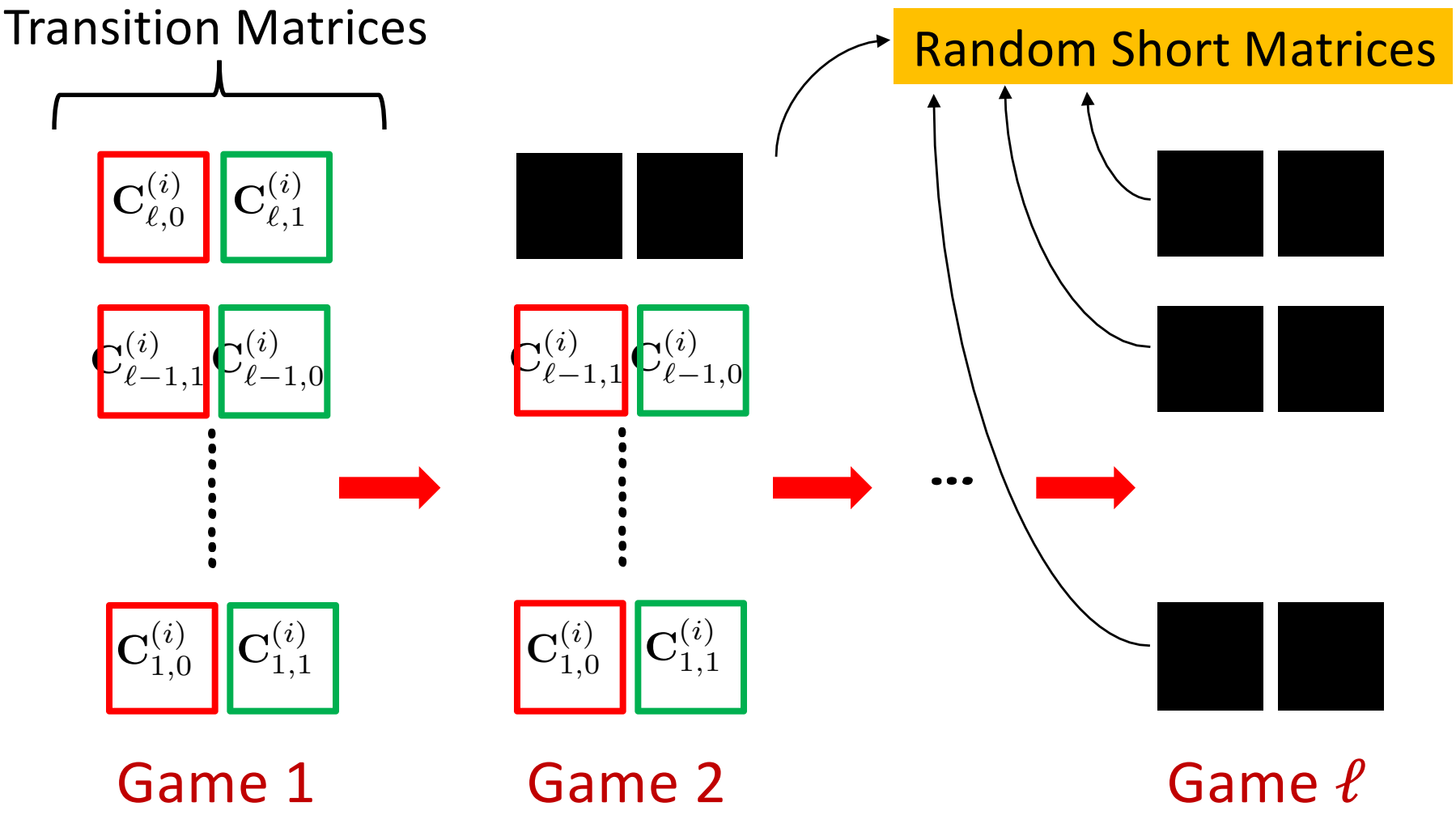
LHL

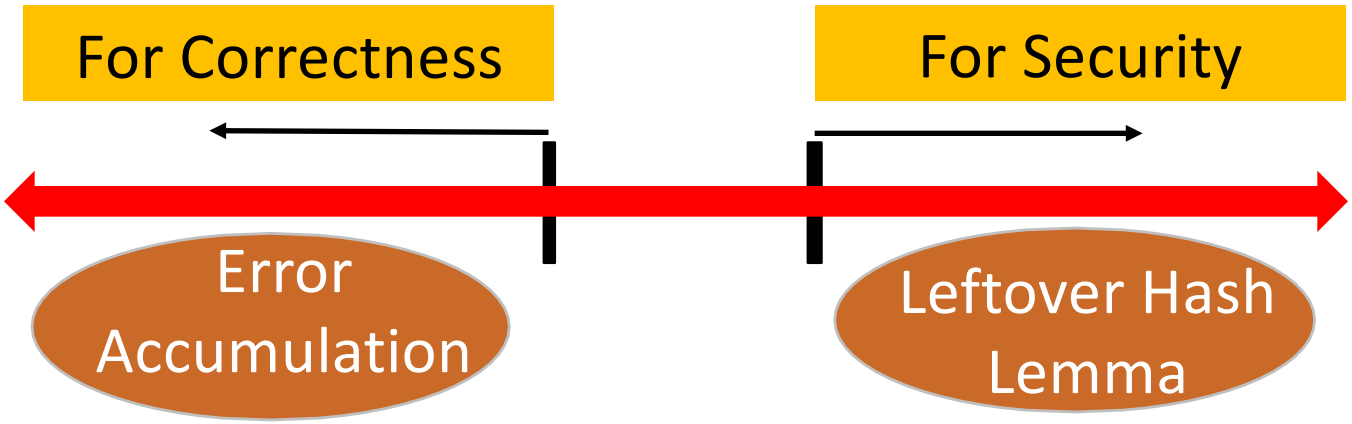
$$\mathbf{M}^{(i)}_{\ell+1,v} \leftarrow \mathbb{Z}_q^{n\times m}$$

Proof Sketch

Trapdoor Properties
LWE



Setting Parameters!?



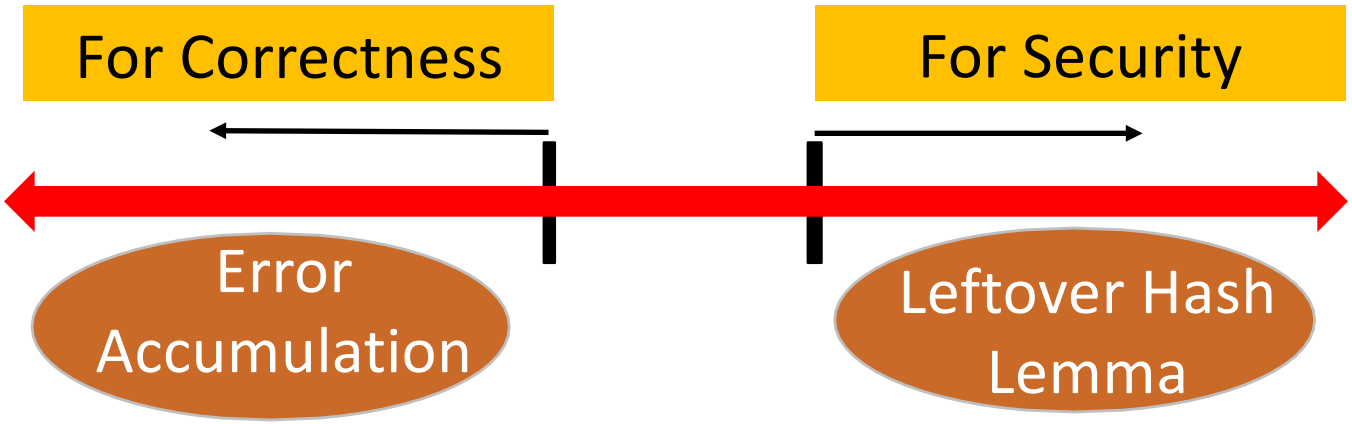
Problem.

For LHL: Output Length **>** $log(q)$.

Error Accumulation: *#Levels* **<** $log(q)$.

Currently: Output Length **=** #Levels.

Setting Parameters!?



Problem.

For LHL: Output Length **>** $log(q)$.

Error Accumulation: *#Levels* **<** $log(q)$.

~~Currently: Output Length = #Levels.~~

Obfuscate PRG(P(·)) instead!

Summing it all up

- Removing fixed-input selector restriction
  - Pad branching programs
- Relieving parameter tension
  - Compose with a NC^1 PRG
- Log-depth => poly-size circuits
  - Use LHE for bootstrapping
- Extending to multi-bit messages
  - See paper

Concluding Remarks

- More applications of lockable obfuscation?
  - [BadrinarayananKhuranaSahaiWaters18]: CCA-secure non-FE compatible scheme
- Stronger notions of obfuscation from standard assumptions?
- Direct construction for circuits and other models of computation?
  - [ChenVaikuntanathanWee18]: Lockable obf. for *non-permutation* BPs

Thanks!