Function privacy for GSW and efficient sign computation from TFHE

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LATCA 2018 — Bertinoro

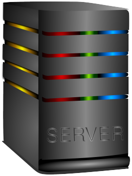
1. FHE Circuit Privacy Almost for Free [BPMW16]

2. Fast Homomorphic Evaluation of Deep Discretized Neural Networks [BMMP18]

Bourse, del Pino, Minelli, Wee

FHE Circuit Privacy Almost for Free

The incentive



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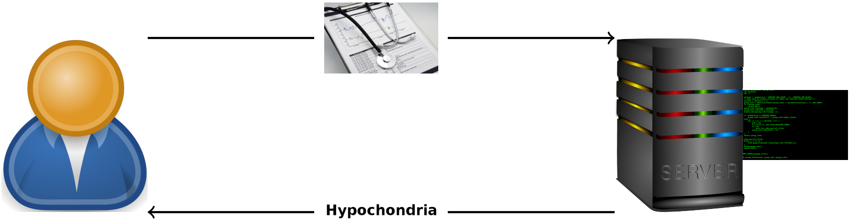

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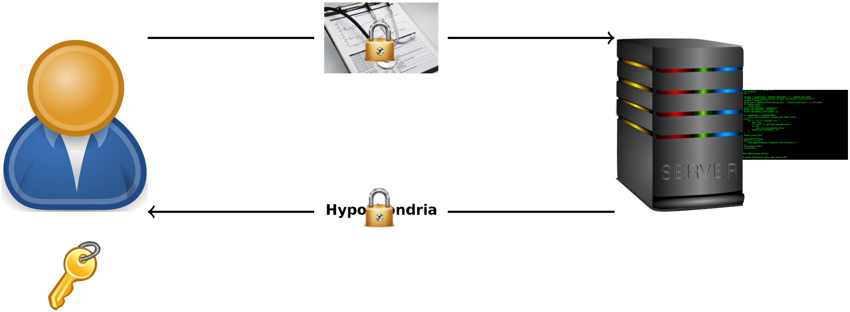

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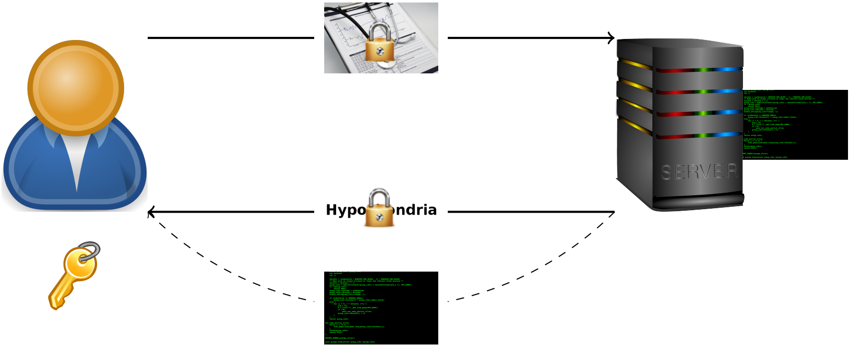

The reminder

LWE

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{s}\mathbf{A} + \mathbf{e} \end{pmatrix} \text{ looks uniform}$$

The reminder

GSW

$$\mathbf{G} = \mathbf{Id}_n \otimes \mathbf{g}, \quad \mathbf{g} = \left(1, 2, \ldots, 2^k\right)$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{A} \\ \mathbf{s}\mathbf{A} + \mathbf{e} \end{pmatrix} + \mu \mathbf{G} \in \mathbb{Z}_q^{n \times m}$$

The reminder

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Sum $\mathbf{C}_1 + \mathbf{C}_2$

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GSW

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Sum $\mathbf{C}_1 + \mathbf{C}_2$

Product $\mathbf{C}_1 \cdot \mathbf{G}^{-1}(\mathbf{C}_2)$

where $\forall\, \mathbf{v} \in \mathbb{Z}_q^n$, $\mathbf{G}^{-1}(\mathbf{v}) \in \mathbb{Z}_q^m$ is *small* and s.t. $\mathbf{G} \cdot \mathbf{G}^{-1}(\mathbf{v}) = \mathbf{v}$

The genius idea

MUX: $\mu, v_0, v_1 \mapsto v_\mu$

$$\mathbf{V}_0 + \mathbf{C} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$$

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$$\mathbf{V}_0 + \mathbf{C} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0) = \mathbf{V}_\mu + \begin{pmatrix} \mathbf{A} \\ \mathbf{s}\mathbf{A} + \mathbf{e} \end{pmatrix} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$$

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- $\mathbf{A} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ looks random (LHL) if $\mathbf{G}^{-1}$ has enough min-entropy

The genius idea

$$\text{MUX: } \mu, v_0, v_1 \mapsto v_\mu$$

$$\mathbf{V}_0 + \mathbf{C} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0) = \mathbf{V}_\mu + \begin{pmatrix} \mathbf{A} \\ \mathbf{sA} + \mathbf{e} \end{pmatrix} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$$

- $\mathbf{A} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ looks random (LHL) if $\mathbf{G}^{-1}$ has enough min-entropy
- Need a way to make $\mathbf{e} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ independent of $\mathbf{V}_1 - \mathbf{V}_0$

The technical part

$$\mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0) = \mathrm{BitDecomp}(\mathbf{V}_1 - \mathbf{V}_0)$$

$\mathbf{e} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ entirely determined

The technical part

$\mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ Gaussian such that $\mathbf{G} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0) = \mathbf{V}_1 - \mathbf{V}_0$

$\mathbf{e} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ Gaussian in a coset of $\mathbf{e} \cdot \Lambda^{\perp}(\mathbf{G})$

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$\mathbf{e} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ Gaussian in a coset of $\mathbf{e} \cdot \Lambda^{\perp}(\mathbf{G})$

Coset still depends on $\mathbf{V}_1 - \mathbf{V}_0$

The technical part

$\mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ Gaussian, $\mathbf{z}$ Gaussian

$\mathbf{e} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0) + \mathbf{z}$ ensures the domain is $\mathbb{Z}$

The technical part

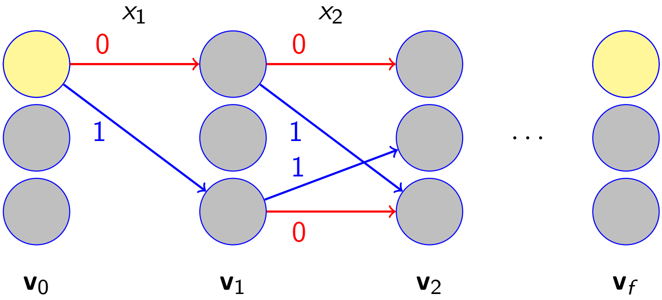
$\mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0)$ Gaussian, $\mathbf{z}$ Gaussian

$\mathbf{e} \cdot \mathbf{G}^{-1}(\mathbf{V}_1 - \mathbf{V}_0) + \mathbf{z}$ ensures the domain is $\mathbb{Z}$

Now only depends on $\|\mathbf{e}\|$

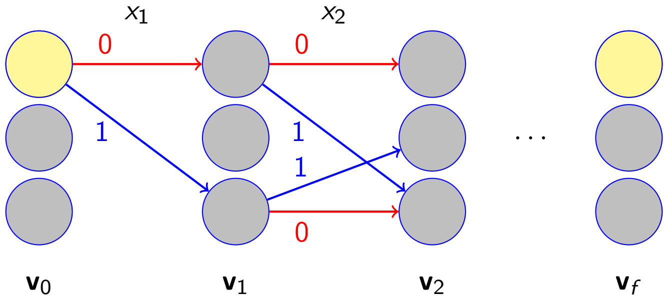
The final result

Branching Program



The final result

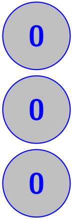
Branching Program



$$\mathbf{v}_{t-1}[i] = \mathrm{MUX}(x_t, \mathbf{v}_{t-1}[j], \mathbf{v}_{t-1}[k])$$

The final result

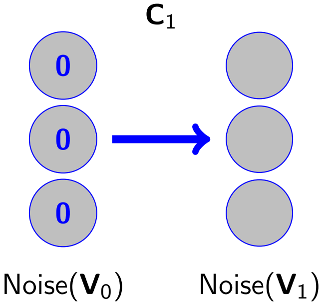
Branching Program



Noise($\mathbf{V}_0$)

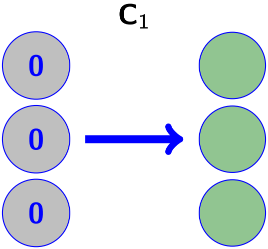
The final result

Branching Program



The final result

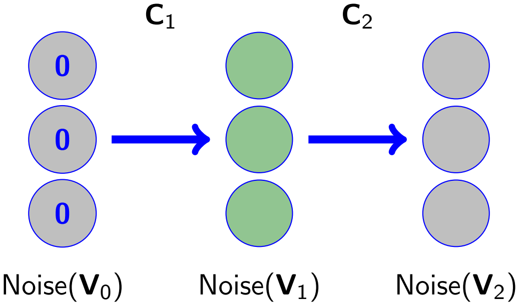
Branching Program



Noise($\mathbf{V}_0$) Noise($\mathbf{V}_1$)

The final result

Branching Program



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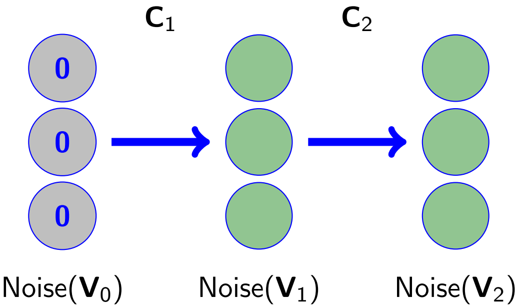
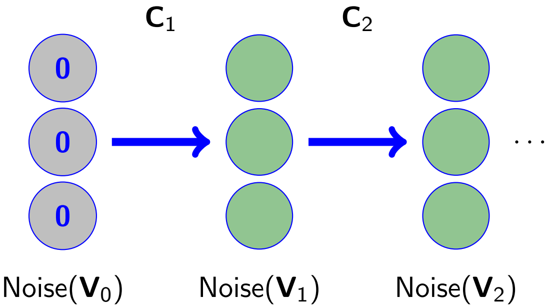


The final result

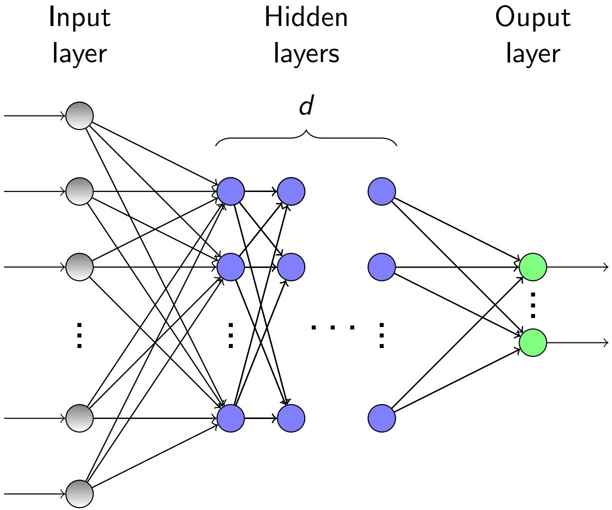
Branching Program



Bourse, Minelli, Minihold, Paillier

Fast Homomorphic Evaluation of Deep Discretized Neural Networks

The problem



The problem

Previous approaches:

The problem

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- use Somewhat Homomorphic Encryption

The problem

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- low degree activation function

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Previous approaches:

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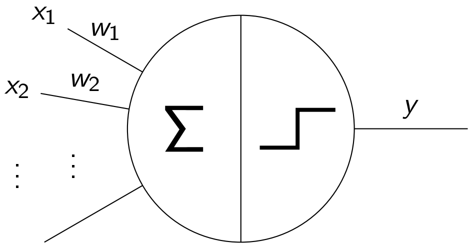
- doesn't scale very well for deep networks :(

Our approach:

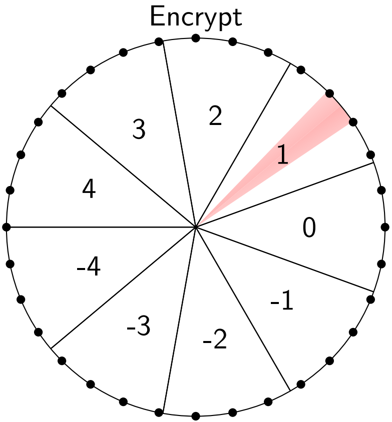
- use efficient bootstrapping techniques
- sign activation function
- evaluation linear in number of neuron :)

The problem



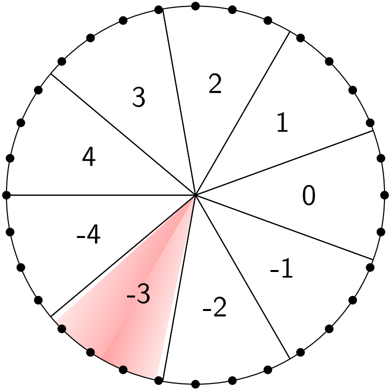
$$y = \text{sign}\left(\sum_i w_i x_i\right)$$

Per neuron evaluation

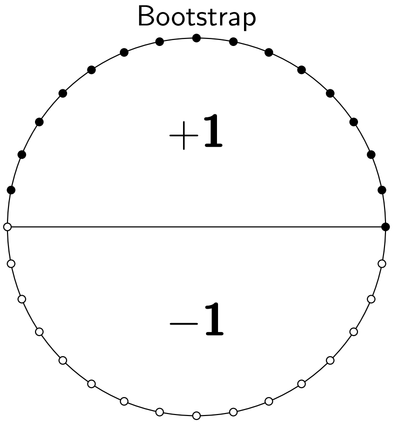


Per neuron evaluation

Homomorphic addition



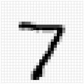
Per neuron evaluation



The big picture

Evaluation of a 784:30:10 neural network for handwritten digit recognition

User



$\xrightarrow{\mathrm{Enc}(\sum_i p_i X^i)}$ 1 RLWE $\xrightarrow{\cdot \sum_i w_i X^{-i}}$ 30 RLWE

Server

30 RLWE

$\downarrow$ Extract

30 N-LWE

$\downarrow$ Key Switching

30 n-LWE

$\downarrow$ Sign Bootstrapping

30 N-LWE

$\downarrow$ weighted sums

10 N-LWE

7 $\xleftarrow{\text{argmax}}$ 10 scores $\xleftarrow{\text{Dec}}$ 10 N-LWE

The results

MNIST dataset, classification of handwritten digits:

	Neurons	ct size	Accuracy	Time enc.	Time eval	Time dec.
Cryptonets	945	586 MB	98.95%	122 s	570 s	5 s
Cryptonets*	945	73.3 kB	98.95%	0.015 s	0.07 s	0.0006 s
FHE-DiNN30	30	≈8.2 kB	93.71%	0.168 ms	0.49 s	0.0106 ms
FHE-DiNN30	100	≈8.2 kB	96.35%	0.168 ms	1.65 s	0.0106 ms